

EXPERIMENTS

EXPERIMENT 1

AIM

To determine resistance per unit length of a given wire by plotting a graph of potential difference versus current.

APPARATUS AND MATERIAL REQUIRED

A wire of unknown resistance ($\sim 10\Omega$), battery eliminator or an accumulator (0 to 3V) or two dry cells (1.5 V each), voltmeter (0-5 V), milliammeter (0-500 mA), rheostat, plug key, connecting wires and a piece of sand paper.

PRINCIPLE

Ohm's law states that the electric current flowing through a conductor is directly proportional to the potential difference across its ends, provided the physical state of the conductor remains unchanged.

If I be the current flowing through the conductor and V the potential difference across its ends, then according to Ohm's law

$$V \propto I$$

and hence

$$V = RI$$

(E 1.1)

where R is the constant of proportionality and is termed as the electrical resistance of the conductor. If V is expressed in volts and I in amperes, then R is expressed in ohms. The resistance R , depends upon the material and dimensions of the conductor. For a wire of uniform cross-section, the resistance depends on the length l and the area of cross-section A . It also depends on the temperature of the conductor. At a given temperature the resistance

$$R = \frac{l}{A}$$

(E 1.2)

where ρ is the specific resistance or resistivity and is characteristic of the material of wire.

Combining Eqs. (E 1.1) and (E 1.2) we have

$$(E 1.3) \quad V = \frac{l}{A} I$$

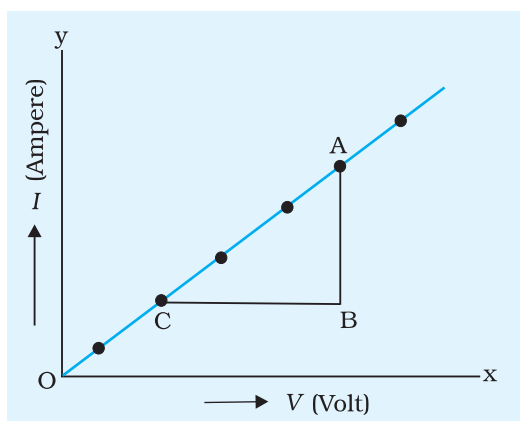


Fig. E 1.1 Graph between current I and potential difference, V

A linear relationship is obtained between V and I , i.e. the graph between V and I will be a straight line passing through the origin as shown in Fig. E 1.1. The slope of the graph is $1/R$ from Eq. (E 1.1) (Equation of straight line passing through origin is $y = mx$ where m is the slope of graph).

$$\text{Slope} = \frac{1}{R}$$

$$\Rightarrow R = \frac{1}{\text{slope}}$$

If l is the length of wire then the resistance per unit length of the wire $\frac{R}{l}$.

PROCEDURE

1. Clean the ends of the connecting wires with the help of sand paper in order to remove any insulating coating on them.
2. Connect various components - resistance, rheostat, battery, key, voltmeter and ammeter as shown in Fig. E 1.2.

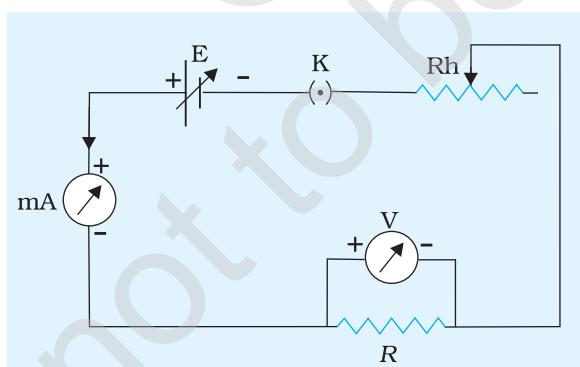


Fig. E 1.2 Circuit to find the relation between current I and potential difference, V for a given wire

3. Note whether pointers in milliammeter and voltmeter coincide with the zero mark on the measuring scale. If it is not so, adjust the pointer to coincide with the zero mark by adjusting the screw provided near the base of the needle using a screw driver.
4. Note the range and least count of the given voltmeter and milliammeter.
5. Insert the key K and slide the rheostat contact to one of its extreme ends, so that current passing through the resistance wire is minimum.
6. Note the milliammeter and voltmeter readings.

- Remove the key K and allow the wire to cool, if heated. Again insert the key. Shift the rheostat contact slightly to increase the applied voltage. Note the milliammeter and voltmeter reading.
- Repeat step 7 for four different settings of the rheostat. Record your observations in a tabular form.

OBSERVATIONS

- Range of ammeter = 0 ... mA to ...mA
- Least count of ammeter = ... mA
- Range of voltmeter = 0 ... V to ...V
- Least count of voltmeter = ...V
- Least count of metre scale = ... m
- Length of the given wire, $l = \dots\text{m}$

Table E 1.1: Voltmeter and milliammeter readings

Sl. No.	Applied potential difference [voltmeter reading V (V)]	Current flowing through the wire [milliammeter reading I (mA)]
	V	I
1		
2		
--		
6		

CALCULATIONS

- Plot a graph between the potential difference across the wire (V) and the current (I) flowing through it as shown in Fig. E 1.1.
- Determine the slope of the graph. The resistance of the given wire is then equal to the reciprocal of the slope.

From the graph $R = \frac{BC}{AB} \dots \Omega$

- Resistance per unit length of given wire = $\frac{R}{l} = \dots \Omega \text{ m}^{-1}$

Error

$$\frac{R}{R} = \frac{V}{V} = \frac{I}{I} = \dots$$

(E 1.4)

Here, R is resistance per unit length and ΔR is estimated error. ΔV and ΔI are the least count of the voltmeter and milliammeter respectively.

RESULT

1. The potential difference across the given wire varies linearly with the current.
2. The resistance per unit length of the wire is $(R \pm \Delta R)$ = (... .. Ωm^{-1}).

PRECAUTIONS

1. The voltmeter should be connected in parallel and the ammeter in series with the circuit. It should be ensured that current enters at the positive terminal and leaves at the negative terminal.
2. The key should be inserted only while taking observations, as excessive flow of current causes unnecessary heating of the wire.
3. Zero error in measuring instruments (voltmeter, ammeter, metre scale) must be taken cognizance of and should be eliminated in case of ammeter and voltmeter by adjusting the pointer with the help of the screw provided at the base of the needle, using a screw driver.

SOURCES OF ERROR

1. The wire used may not be of uniform area of cross-section.
2. The length of the resistance wire measured should be between one terminal of voltmeter and the other. The lengths of ends wound around the terminals of voltmeter, if included, would give error in measured length.

DISCUSSION

A resistor obeys Ohm's law. However, not all conducting devices obey Ohm's law e.g. diode, thyristor etc. These are called non-ohmic resistances.

SELF ASSESSMENT

1. A voltmeter is always connected in parallel and an ammeter in series with the circuit. Why? Will they record the respective parameters if connected in opposite manner?

- Why are copper wires normally used for connecting different components in an electric circuit?
- What happens if the current is allowed to flow through the circuit continuously for a long time? Why?

SUGGESTED ADDITIONAL EXPERIMENTS/ACTIVITIES

- Study the effect of length of wire on its resistance when its diameter is kept constant.
- Study the effect of diameter of wire on its resistance when its length is kept constant by using different SWG (standard wire gauge) wires available in the market.
- Study the resistivity of wires made up of different materials. Is the resistivity of all wires same?
- Two uniform wires of the same material have the same length. One has circular cross section of area A while the other is not circular but has the same area of cross section A . Will their resistances be equal?
- Study the voltage current relationship for the filament of a torch bulb.
- Set up a circuit as shown in Fig. E 1.3.

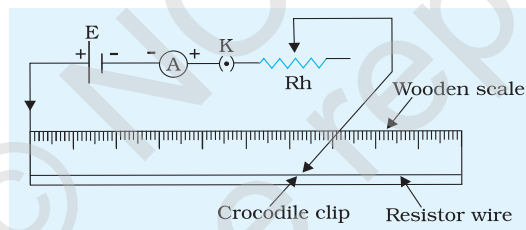


Fig. E 1.3

Note the ammeter reading I by connecting the crocodile clip at 10, 20, 30 cm length of wire. Plot a graph between $1/I$ and l . Find its slope and interpret its result. Can you use the graph to check the homogeneity of the resistance wire?

- Four resistances R_1 , R_2 , R_3 and R_4 are connected together as shown in Fig. E 1.4. Complete the electric circuit by connecting the combination of resistances to a 6 V source, an ammeter and a voltmeter. Explain, how would you measure current/voltage flowing through each one of the resistors.

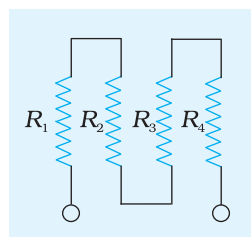


Fig. E 1.4

EXPERIMENT 2

AIM

To determine the resistance of a given wire using a metre bridge and hence determine the resistivity of the material of the wire.

APPARATUS AND MATERIAL REQUIRED

Metre bridge, a wire about 1m long (of material whose specific resistance is to be determined), a resistance box, a rheostat, galvanometer, a jockey, one-way key, a cell or battery eliminator, thick connecting wires, sand paper, screw gauge.

DESCRIPTION OF APPARATUS

Metre bridge

It consists of one metre long constantan wire AC of uniform cross-sectional area mounted on a wooden board with a scale (Fig. E 2.1). The two ends of the wire are attached to terminals A and C. Thick metal strips bent at right angles are used to provide two gaps E and F to connect resistors forming a Wheatstone's

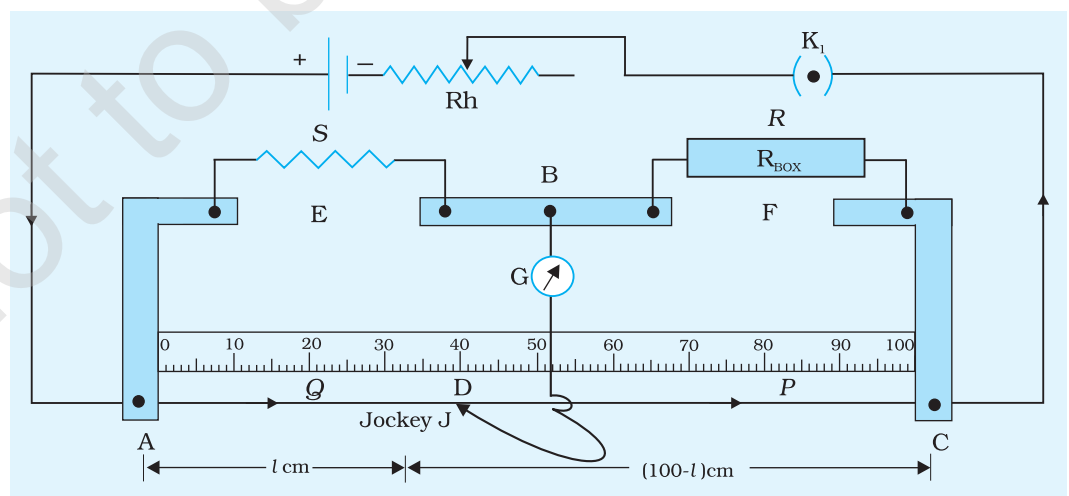


Fig. E 2.1 A metre bridge

bridge (Fig. E 2.2). The terminal B between the gaps is used for connecting galvanometer and other end of the galvanometer is connected to a jockey J.

PRINCIPLE

A metre bridge works on the principle of Wheatstone's bridge. As shown in Fig. E 2.2, it consists of four resistors **P**, **Q**, **R** and **S** connected in the form of a network ABCD. The terminals A and C are connected to two terminals of a cell through a key K_1 . Terminals B and D are connected to a sensitive galvanometer G through a key K_2 .

If there is no deflection in the galvanometer G, then balance condition for Wheatstone's bridge is

$$\frac{P}{Q} = \frac{R}{S} \quad \text{(E 2.1)}$$

We use this relation (E 2.1) to determine **S** if **P**, **Q** and **R** are known.

The unknown resistance **S** is connected in the gap E and a resistance box (R_{Box}) in gap F of the metre bridge. The terminal B is connected to one terminal of the galvanometer G. The other terminal of the galvanometer is connected to a jockey J which slides along the wire AC. A source of dc current is connected between A and C through a key K_1 so as to provide a constant potential drop along AC (Fig. E 2.1).

A resistor (or wire) of known resistance is inserted in the gap F by taking out corresponding key from the resistance box R_{Box} . The jockey is moved on the wire AC to obtain a condition of no-deflection in the galvanometer. It happens when the jockey is kept at a point D called the null point. In this condition;

$$\frac{P}{Q} = \frac{R}{S} = \frac{\text{Resistance of wire of length DC}}{\text{Resistance of wire of length AD}} \quad \text{(E 2.2)}$$

Unknown resistance **S** of the wire, having uniform cross-sectional area, is then given by

$$S = R \frac{l}{100 - l} \quad \text{(E 2.3)}$$

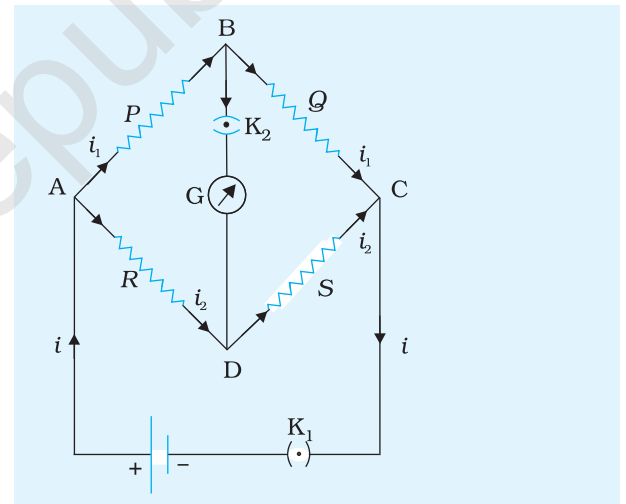


Fig.E 2.2 The Wheatstone's bridge

Because, for a wire of uniform cross-sectional area, resistance is proportional to length.

Thus, knowing l and R , and using Eq. (E 2.3), the unknown resistance S can be determined.

Resistivity

The specific resistance or resistivity ρ of the material of the given wire is

where S is the resistance of the wire of length L and $a = \pi r^2$, (r being the radius) is the area of cross-section.

PROCEDURE

1. Find the average diameter of the wire with a screw gauge. From this, obtain the value of its radius r .
2. Clean the insulation at the ends of connecting wires with a piece of sand paper. Tighten all plugs of the resistance box (R_{BOX}) by pressing each plug.
3. Set up the circuit as shown in Fig. E 2.1 with unknown resistance wire of known length in gap E.
4. Next, introduce some resistance R in the circuit from the resistance box. Bring the jockey J in contact with terminal A first and then with terminal C. Note the direction in which pointer of the galvanometer gets deflected in each case. Make sure that jockey remains in contact with the wire for a fraction of a second. If the galvanometer shows deflection on both sides of its zero mark for these two points of contact of the jockey, null point will be somewhere on the wire AC. If it is not so, adjust resistance R so that the null point is somewhere in the middle of the wire AC, say, between 30 cm and 70 cm.
5. If there is one-sided deflection, check the circuit again, especially junctions, for their continuity.
6. Repeat step 4 for four different values of resistance R .
7. Interchange the position of the resistances S and R and repeat steps 4 to 6 for the same five values of R . While interchanging S and R , ensure that the same length of wire of resistance S is now in the gap F. The interchange takes care of unaccounted resistance offered by terminals.

OBSERVATIONS

1. Length of the wire of unknown resistance, $L = \dots \text{cm}$
2. Measurement of diameter of wire of unknown resistance

Least count of the screw gauge (L.C.) = ... mm

Zero error of the screw gauge = ... mm

Zero correction of the screw gauge = ... mm

Table E 2.1: Diameter of Wire

Sl. No.	Reading along one direction			Reading along mutually perpendicular direction			Mean Diameter
	Main scale reading p (mm)	Coinciding circular scale division n	Diameter $d_1 = p + n \text{ L.C.}$ (mm)	Main scale reading p' (mm)	Coinciding circular scale division n'	Diameter $d_2 = p' + n' \text{ L.C.}$ (mm)	
							$d = \frac{d_1 + d_2}{2}$ (mm)
1							
2							
3							

Mean diameter (corrected for zero error) = ... mm

Radius of wire $r = \dots \text{mm}$

Table E 2.2: Unknown resistance

Sl. No.	S in the left gap				S in the right gap				Mean resistance	ρ $\Omega \text{ m}$	ΔS_1 Ω	ΔS_2 Ω	ΔS Ω	Δp $\Omega \text{ m}$
	Resistance R Ω (ohm)	Position of balance point D (cm)	Balancing length AD l (cm)	Length DC = $100 - l$ (cm)	Position of balance point D' (cm)	Balancing length AD' = l' (cm)	Length D'C' = $100 - l'$ (cm)	$100 \frac{l}{l'}$ (ohm)						
1														
2														
...														
5														

Mean

CALCULATIONS

$$L = \dots \text{ cm} \quad r = \dots \text{ mm} \quad S = \dots \Omega$$

Substituting these values in $\rho = S \frac{\pi r^2}{L}$, calculate the value of ρ in ohm-m.

Error

$$(E 2.4) \quad \frac{S}{S} \frac{2r}{r} \frac{L}{L}$$

Errors Δr , ΔL are the least counts of the measuring instruments and error ΔS is the maximum of the values obtained by following equations :

$$(E 2.5) \quad S_1 \frac{l}{l} \frac{l}{100 l} S_1$$

$$(E 2.6) \quad S_2 \frac{l}{l} \frac{l}{100 l} S_2$$

If it is presumed that resistance offered by the resistance box is the same as that indicated on it, then $\Delta R = 0$

Therefore, maximum error is $\Delta S = \Delta S_1 + \Delta S_2$

RESULT

1. The unknown resistance of the given wire is found to be $S + \Delta S = \dots \dots \Omega$
2. The resistivity of the material of the wire is $\rho \pm \Delta \rho = \dots \dots \Omega \text{ m}$
Here S and ρ are mean values. ΔS and $\Delta \rho$ are maximum of the five values of error.

PRECAUTIONS

1. All the connections and plugs should be tight.
2. Jockey should be moved gently over the metre bridge wire.
3. The plug in the key (K_1) should be inserted only at the time of taking observations.
4. Null points should be in the middle of the wire (30 cm to 70 cm).

SOURCES OF ERROR

1. The metre bridge wire may not be of uniform area of cross-section.
2. Effect of end resistances due to copper strips, connecting screws, may affect the measurement.
3. The length L of the wire should not include the lengths below the terminals when placed in gap E or F.
4. The resistances of end pieces/metal strips may not be negligible. The error introduced by it can be reduced by interchanging the known and unknown resistances in gaps E and F.
5. The length measurements l and l' may have error if the metre bridge wire is not taut and along the scale in the metre bridge.
6. Galvanometer pointer is expected to be at zero when no current flows through it. However, many times it is observed that it is not so. In such cases, pointer has to be adjusted to zero by gently moving the screw below the scale with the help of a screw driver. Otherwise null point must be obtained by tapping the jockey on the wire.

DISCUSSION

1. It is generally preferred that R and S combination should be so chosen that the balance point comes near the middle of the metre bridge wire. Why? Is the sensitivity of detecting balance point best when R and S have same order of resistance?
2. The accuracy with which the null point can be detected also depends upon the sensitivity of the galvanometer used. To investigate this, find the distance through which the jockey has to be moved to cause a just perceptible deflection of the galvanometer. How can the error due to this be minimised? Does it have something to do with the potential difference between A and C?
3. The length l , that is, the position of the jockey, may be read upto, 0.1 cm (say). How much uncertainty will be introduced in the result due to this error?
4. It is advised that key should be unplugged when readings are not being taken to avoid unnecessary heating of the wire. Why? In what way, will the heating affect the null point? Will it be significant?

SELF ASSESSMENT

1. The bridge wire may not be exactly 100 cm long, so record its exact length and use this in the calculation.
2. If the metre bridge wire is not of uniform area of cross section, how will it affect the observations?

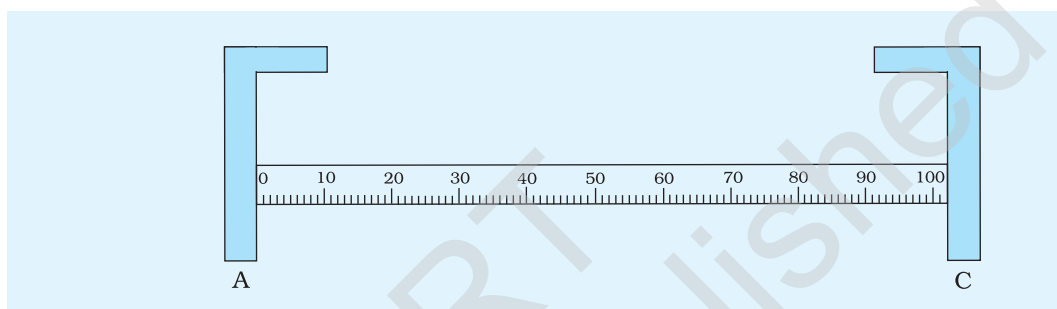


Fig. E 2.3 Defective scale of a metre bridge

3. If the metre bridge wire is not made of a material of uniform density, how will it affect the observations?
4. If the same experiment is performed with $AC = 50$ cm wire instead of 1 m, what changes would be there in the result?
5. The scale attached along the wire may not indicate the exact length as shown in Fig. E 2.3. How would you minimise the error?
6. Why is the metre bridge not suitable for measuring very high/very low resistances?
7. Why is the metre bridge preferred for determining the resistance of a wire over the Ohm's law circuit?

SUGGESTED ADDITIONAL EXPERIMENTS/ACTIVITIES

1. Determine the resistance of an electric bulb of given voltage and power. Attach crocodile clips at the end of two copper wires. Use these to connect the bulb in the gap E as shown in Fig. E 2.4. Why does the resistance obtained by you differ from the theoretical value?
2. Making use of your observations plot a graph between $(100 - l)/l$ and R . Find the slope of the graph. What does it represent?
3. The resistance of a wire changes with temperature. You may study this by using a metre bridge. You may use a beaker with mustard oil, a suitable thermometer (0-300°C) and resistance wire of known length. The experimental set-up shown in Fig. E 2.5 may be used.

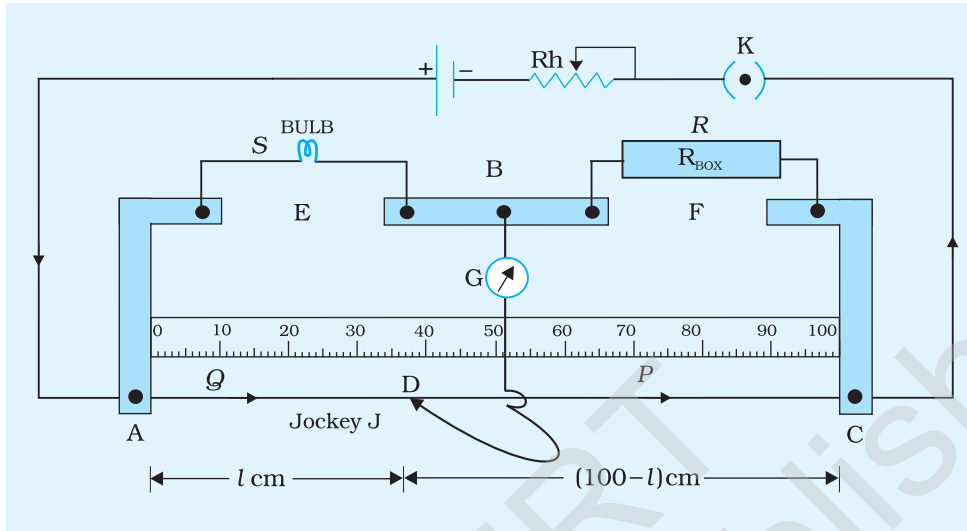


Fig. E 2.4

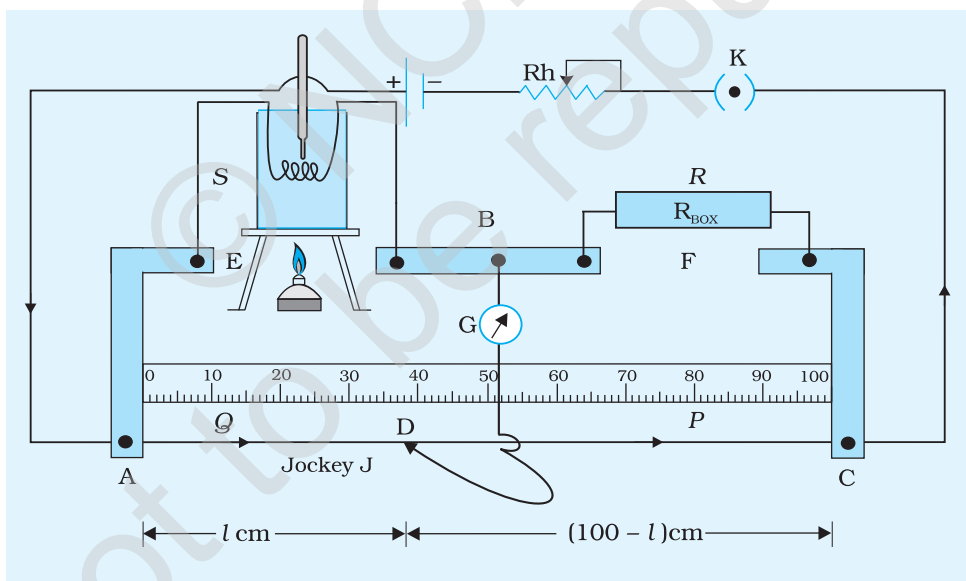


Fig. E 2.5

EXPERIMENT 5

AIM

To determine the internal resistance of a given primary cell using a potentiometer.

APPARATUS AND MATERIAL REQUIRED

Potentiometer, Leclanche cell or dry cell, an ammeter, one resistance box ($R_{\text{BOX } 1}$) (about 0-50 Ω), 3 one way plug keys, galvanometer, a high resistance box ($R_{\text{BOX } 2}$) (about 0-10 k Ω), a low resistance rheostat of about 20 Ω , jockey, lead accumulator and connecting wires.

PRINCIPLE

When a resistance R is connected across a cell of emf E and internal resistance r , then the current I in the circuit is

$$I = \frac{E}{R + r} \quad (\text{E 5.1})$$

The potential difference $V (= RI)$ across the two terminals of the cell is

$$V = \frac{E}{R + r} R \quad (\text{E 5.2})$$

Thus
$$\frac{E}{V} = 1 + \frac{r}{R}$$

or
$$r = \left(\frac{E}{V} - 1 \right) R \quad (\text{E 5.3})$$

If l_0 and l are the distances of the balance null point from end A of the potentiometer for an open and a closed circuit respectively (Fig. E 5.1), then E is proportional to l_0 and V is proportional to l .

$$\frac{E}{V} = \frac{l_0}{l} \quad (\text{E 5.4})$$

From Eqs. (E 5.3) and (E 5.4)

$$(E 5.5) \quad r = \frac{l_0}{l} R$$

PROCEDURE

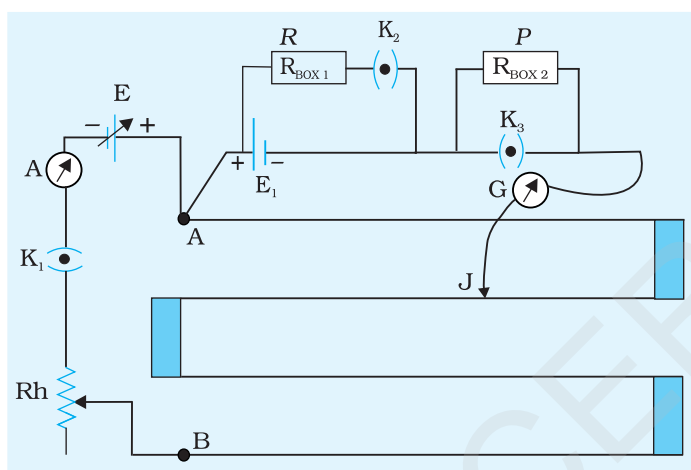


Fig. E 5.1 Circuit to measure internal resistance of a primary cell using a potentiometer

1. Connect different electrical components as shown in the circuit (Fig. E 5.1). After checking the circuit connections, close key K_1 .
2. With keys K_2 and K_3 open and a protective high resistance P from the $R_{\text{BOX } 2}$, find the position of the balance point. For final reading, short circuit the resistance P by closing the key K_3 and find the balance length l_0 .
3. Take $R = 10 \Omega$ (from $R_{\text{BOX } 1}$), close the key K_2 and quickly measure the new balance length l . Open K_2 as soon as this has been done.
4. Keep the readings in the ammeter constant throughout the above observation.
5. Reduce the value of R in equal steps of 1Ω and for each value of R obtain the balance length l .
6. At the end of the experiment, open key K_2 and repeat step 2 to find l_0 again.

OBSERVATIONS

$l_0 = \dots$ cm (in the beginning of the experiment)

$l_0 = \dots$ cm (at the end of the experiment)

Mean $l_0 = \dots$ cm.

Table E 5.1: Balance length

Sl. No.	$R \Omega$	l cm	$\frac{1}{R} \Omega^{-1}$	$\frac{1}{l} \text{ cm}^{-1}$	$r = \left(\frac{l_0 - l}{l} \right) R \Omega$
1					
2					
--					
6					

CALCULATIONS

1. Substitute the value of l_0 , l and corresponding value of R in Eq. (E 5.5) and calculate the value of r , where $r = \frac{l_0 - l}{l} R$.
2. Use graphical method also to obtain r . Note that Eq (E 5.5) can be written as

$$\frac{1}{R} = \frac{l_0}{r} \frac{1}{l} - \frac{1}{r} \quad \text{(E 5.6)}$$

It is an equation of a straight line (Fig. E 5.2).

3. Plot a graph between $1/R$ and $1/l$, taking $\frac{1}{l}$ on the x-axis and $\frac{1}{R}$ on y-axis.
4. Draw a straight line as close to the points plotted as you can. The negative intercept on the y-axis gives the value of $1/r$. Hence, obtain the value of r (Fig. E 5.2).

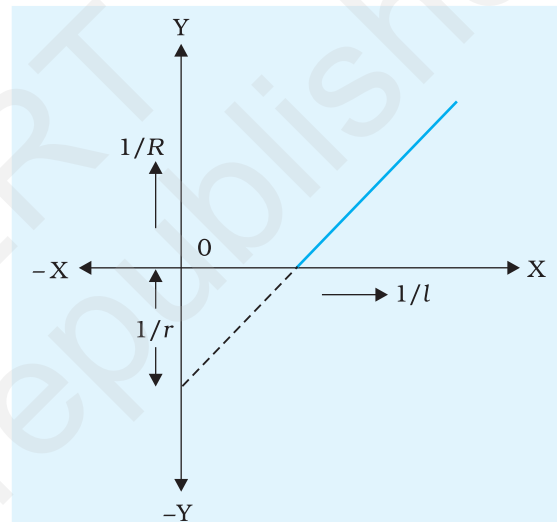


Fig. E 5.2 Graph between $1/R$ and $1/l$

RESULT

The internal resistance of the given cell r

- (i) by calculation ... Ω
- (ii) by graph ... Ω

PRECAUTIONS

1. The primary cell whose internal resistance is to be determined should not be disturbed during the experiment or else its internal resistance may change.
2. The emf of battery E should be more than the emf of the primary cell, E_1 .
3. Positive terminals of E and E_1 both should be connected at the same point on the potentiometer.
4. Always measure length from point A i.e. the point at which positive terminals of battery are connected and measure this length upto the balance point.

5. Insert K_1 and K_2 only when readings are taken otherwise the wires may get heated up due to continuous flow of current and may also affect the internal resistance of the cell.

SOURCES OF ERROR

1. Potentiometer wire may not be of uniform cross - section.
2. Brass strips at the ends may have a finite resistance.
3. Emf of the auxiliary battery producing the drop of potential along the wire may not be constant throughout the course of the experiment.
4. Heating of the potentiometer wire by current may introduce some error.

DISCUSSION

1. The theory of potentiometer assumes that there is a steady current in wire AB during the period of experiment. Therefore, emf of the accumulator should be constant during the course of the experiment.
2. The position of the jockey can be read within the least count of the measuring scale $\pm 0.1\text{cm}$. Moreover, the edge of the jockey may further limit this least count. It is therefore advised to use a sharp edged jockey.
3. There may also be a zero error in the measurement of l , due to the end of the scale not being exactly at the end of the wire.

SELF ASSESSMENT

1. All the positive terminals of sources of emf are joined at point A of potentiometer, but if all the negative terminals of sources of emf are joined at point A then how will the balance length be affected?
2. Find the internal resistance of a freshly prepared Leclanche cell. Does its internal resistance change with R ?
3. State the factors on which the internal resistance of a cell depends.

SUGGESTED ADDITIONAL EXPERIMENTS/ACTIVITIES

1. Find the internal resistance of dry cells of different makes.
2. Can the internal resistance of a secondary cell be determined by this method? Give reason for your answer.

EXPERIMENT 10

AIM

To find the focal length of a convex lens by plotting graphs between u and v or between $1/u$ and $1/v$.

APPARATUS AND MATERIAL REQUIRED

An optical bench, two sharp-edged needle (pins), convex lens of less than 20 cm focal length, three uprights (with clamps), index needle (may be a knitting needle), metre scale and spirit level.

TERMS AND DEFINITIONS

1. **Principal axis** of a lens is the line joining centre of curvature of the two surfaces.
2. **Optical centre** is the point, through which a ray passes undeviated through the lens.
3. **Principal focus** is the point where rays parallel to the principal axis focus after passing through the lens (convex) or appear to come from after passing through the lens (concave).
4. **Focal length** is the distance between optical centre of lens and focus.
5. **Intercepts of a graph:** If a graph cuts x-axis and y-axis, then lengths between origin and points of interception are intercepts of the graph.

GRAPHICAL METHOD FOR FINDING THE POSITION OF AN IMAGE FORMED BY THIN LENSES

The image formed by a thin lens can be located using graphical method by considering refraction of rays emanating from each point on the object. However, it is convenient to choose any two of the following three rays (Fig. E 10.1).

1. A ray from the tip of the object parallel to the principal axis of the lens that passes through the second principal focus F' (in a convex lens) or appears to diverge (in a concave lens) from the first principal focus F after refraction.
2. A ray from the tip of the object incident at the optical centre, passes through the lens undeviated. This is because near its centre, the lens behaves like a thin glass slab.
3. A ray of light from the same point on the object that passes through the first principal focus F (for a convex lens) or appearing to pass through F' (for a concave lens) emerges parallel to the principal axis after refraction.

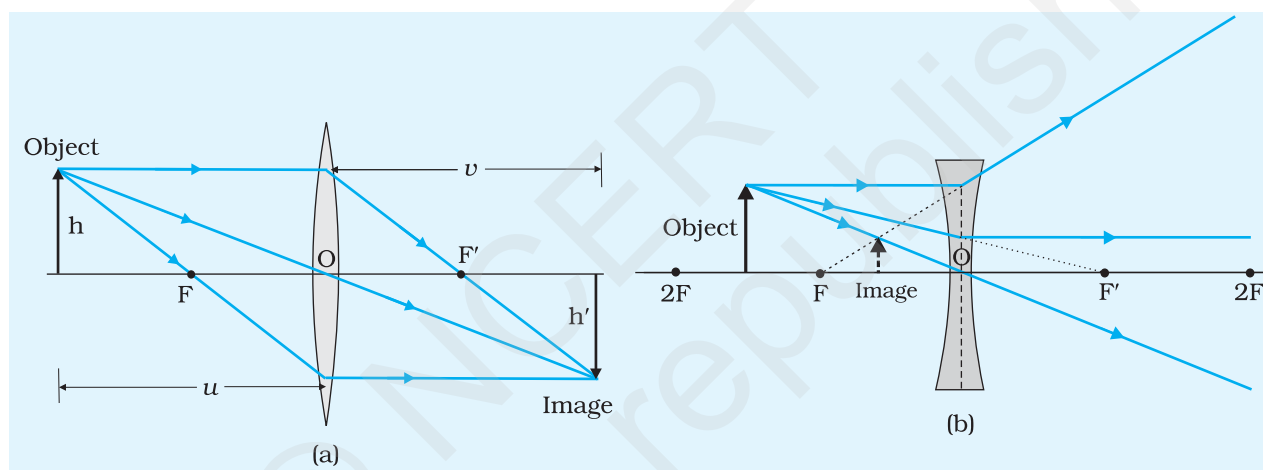


Fig. E 10.1 Ray tracing for locating the image formed by (a) convex lens and (b) concave lens.

PRINCIPLE

For an object placed at a distance u from the optical centre of a thin convex lens of focal length f , a real and inverted image is formed on the other side of the lens at a distance v from the optical centre. The relation between these distances is:

(E 10.1)

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

According to the new cartesian sign convention (*see Physics Textbook, NCERT, 2007, Class XII, Part-II p. 311,*) u is negative but v is positive [Fig. E 10.2 (a) and (b)]. Therefore the Eq. (E 10.1) takes the following form for magnitudes of u and v .

(E 10.2)

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

or, $f = \frac{uv}{u+v}$

(E 10.3)

In this result the positive values of u and v are substituted.

Eq. (E 10.2) shows that $\frac{1}{v}$ versus $\frac{1}{u}$ graph is a straight line of negative

slope. If $\frac{1}{v}$ equals zero or $\frac{1}{u}$ equals zero, then respectively $\frac{1}{u} = \frac{1}{f}$

or $\frac{1}{v} = \frac{1}{f}$. The intercepts of

the graph on both axes are

$\frac{1}{f}$. Graph of u versus v is a

hyperbola. When $u = v$, then each equals $2f$. Eq. (E 10.3) shows that values of u and v are interchangeable.

When an object (say, a pin) is placed in front of a thin convex lens at a distance equal to $2f$, a real and inverted image of same size as that of the object is formed at a distance equal to $2f$ on the other side of the lens [Fig. E 10.2(a)]. If the object's position lies in between distance $2f$ and distance f from the optical centre of the lens then a real, inverted and magnified image is formed at a point beyond $2f$ from the optical centre on the other side of the lens [Fig. E 10.2(b)].

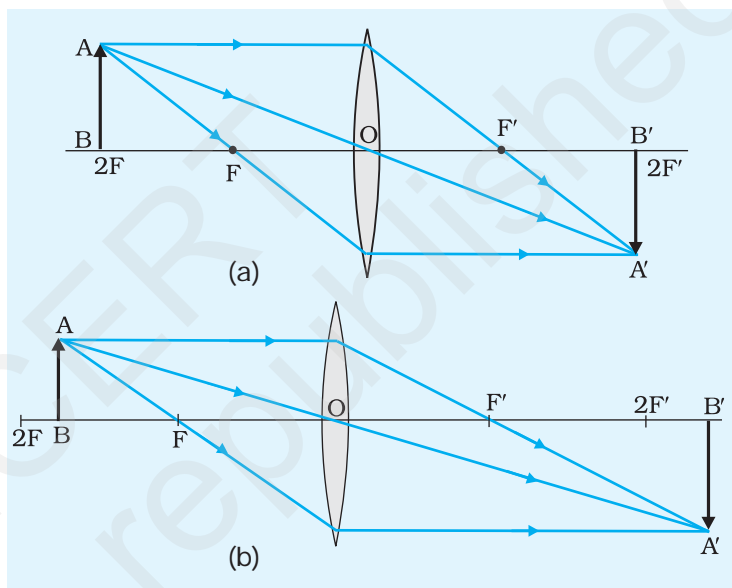


Fig. E 10.2 (a), (b) Formation of image by a convex lens (a) $u = 2f$ and (b) $2f > u > f$.

Thus, by measuring the distances u and v , the focal length of the convex lens can be determined using Eq. (E 10.3). The focal length of the lens may also be determined by plotting graphs between u and v or between $1/u$ and $1/v$.

PROCEDURE

1. Obtain approximate value of the focal length of the thin convex lens by focusing the image of a distant object. It can be found by obtaining a sharp image of the Sun or a distant tree on a screen, say a plane wall, or a sheet of paper placed on the other side of the lens and measuring the distance between the lens and the image with a scale. This distance is a rough estimate of the focal length, f of the convex lens.

Note: Do not look at the image of Sun directly as it may hurt your eyes.

- Place the optical bench on a rigid table or on a platform, and using the spirit level, make it horizontal with the help of levelling screws provided at the base of the bench.
- Clamp the convex lens on an upright and mount it vertically almost near to the middle of the optical bench such that its principal axis is parallel to the optical bench. In this position, the lens would lie in a plane perpendicular to the optical bench.
- For the determination of the index correction, bring a mounted pin close to the lens. Adjust the index needle (a sharp-edged knitting needle would also serve the purpose) horizontally such that its one end touches one of the curved surfaces of the lens and the other end touches the tip of the pin. Note the positions of the two uprights on the scale provided on the optical bench. The difference of the two would give the **observed length** of the index needle. The **actual length** between the tip of the pin and optical centre O would be length of the index needle (as measured by a scale) plus half of the thickness of the lens because optical centre of a double convex lens with surfaces of equal curvature is at its geometrical centre. The difference of the two lengths is the **index correction**. Find index correction for both the pins.
- Place the vertically mounted sharp pins P and P' (Fig. E 10.3)

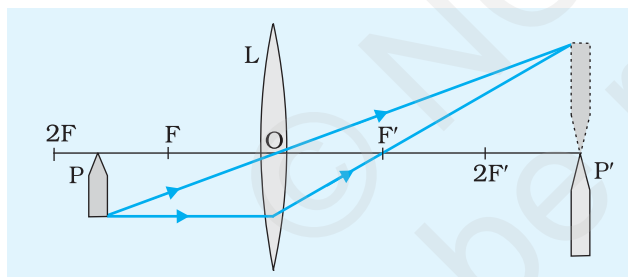


Fig. E 10.3 The ray diagram for finding the focal length of a convex lens.

on left and right hand sides of the lens respectively. Adjust the pins P and P' so that the heights of the tips of these pins become equal to the height of the optical centre O of the lens from the base of the optical bench. Let the pin P (placed on left hand side of the lens) be the object pin and the pin P' (lying on right hand side) be the image pin. Put a small piece of paper on one of the pins (say on image pin P') to differentiate it from the object pin P'.

- Displace the object pin P (on left side of the lens) to a distance slightly less than $2f$ from the optical centre O of the lens (Fig. E 10.3). Locate the position of the real and inverted image on the other side of the lens above the image pin P'.
- Using the method of parallax, adjust the position of the image pin P' such that the image of the object pin P coincides with the image pin P'.

Note: As the value of u changes from $2f$ to f , v changes from $2f$ to infinity. Since the values of u and v are interchangeable, i.e., the object and image are two conjugate points, therefore it is clear that complete range of values for both u and v between f and infinity are obtained for a movement of the object pin over the range $2f$ to f .

8. Note the upright position of the object pin, convex lens and image pin on the optical bench and record the readings in an observation table.
9. Move the object pin P closer to the optical centre O of the lens (say by 2 cm or 3 cm). Repeat the experiment and record at least six sets of readings for various distances of object pin between f and $2f$ from the lens.

OBSERVATIONS

1. Approximate focal length of the convex lens = ... cm
2. Length of the index needle as measured by the metre scale, $L_o = \dots$ cm
3. Thickness of the thin convex lens (given), $t = \dots$ cm
4. Actual length between the optical centre O of the lens and tip of the pin, $l_o = L_o + t/2 = \dots$ cm
5. Observed length of the index needle, $l'_o =$ Distance between the centre of convex lens and tip of the object pin
 = Position of lens upright – position of object pin upright on the scale.
 = ... cm – ... cm = ...cm
6. Index correction for object distance, $e_o = l_o - l'_o = \dots$ cm; similarly for image pin, $e_i = l_i - l'_i = \dots$ cm

Table E 10.1: Determination of u , v and f

Sl. No.	Lens upright a (cm)	Object pin upright b (cm)	Image pin upright c (cm)	Observed $u = a - b$ (cm)	Observed $v = a - c$ (cm)	Corrected $u =$ Observed $u + e_o$ (cm)	Corrected v $=$ Observed $v + e_i$ (cm)	$\frac{1}{u}$ cm^{-1}	$\frac{1}{v}$ cm^{-1}	f $\frac{uv}{u+v}$ cm	Δf (cm)
1											
2											
...											
6											
Mean											

CALCULATIONS

- A. Calculate the corrected values of u and v . Compute the value of f using Eq. (E 10.3). Tabulate them in the table and find the mean value of the focal length of the given convex lens.

Error

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\text{or } \frac{\Delta f}{f^2} = \frac{\Delta u}{u^2} + \frac{\Delta v}{v^2}$$

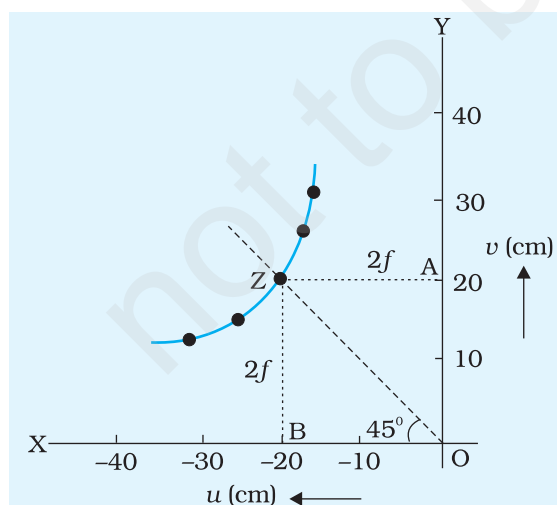
$$\Delta f = f^2 \left[\frac{\Delta u}{u^2} + \frac{\Delta v}{v^2} \right]$$

Maximum of six values of Δf is to be reported with the result as the experimental error.

CALCULATION OF FOCAL LENGTH BY PLOTTING GRAPHS

(A detailed method of plotting graphs has been illustrated in Chapter 1 Article 1.8 (p. no. 15).

- B. **$u - v$ Graph:** Take u along x-axis and v along y-axis. Scales of x- and y-axis should be same. Draw a hyperbola curve for various values of u and v (Fig. E 10.4). Note that six sets of readings for u between f and $2f$, give you 12 points on the graph by interchanging values of u and v .



The point $u = 2f$; $v = 2f$ is shown as point Z on $u - v$ graph (Fig. E 10.4). The point Z is the point of intersection of a line OZ bisecting the angle $\angle XOY$ with hyperbola. Draw two lines AZ and BZ perpendicular to Y- and X-axis, respectively. The lengths AZ and BZ are both equal to distance $2f$. Thus by plotting the $u - v$ graph, the focal length of the lens can be obtained.

Distance OA (= $2f$) on y-axis = ... cm

Distance OB (= $2f$) on x-axis = ... cm

Mean focal length of the convex lens, f

Fig. E 10.4 u versus v graph for convex lens = $\frac{OA+OB}{4} = \dots$ cm

C. $1/u - 1/v$ graph: Draw a straight line graph by plotting $1/u$ along the X-axis and $1/v$ along the Y-axis (Fig. E 10.5). Both the intercepts OA' (on y-axis) and OB' (on X-axis) will be equal to distance $1/f$.

Intercept OA' ($= 1/f$) on y-axis
 $= \dots \text{cm}^{-1}$

Intercept OB' ($= 1/f$) on x-axis
 $= \dots \text{cm}^{-1}$

Mean focal length (f) of the convex

$$\text{lens} = \frac{2}{OA' + OB'} = \dots \text{cm.}$$

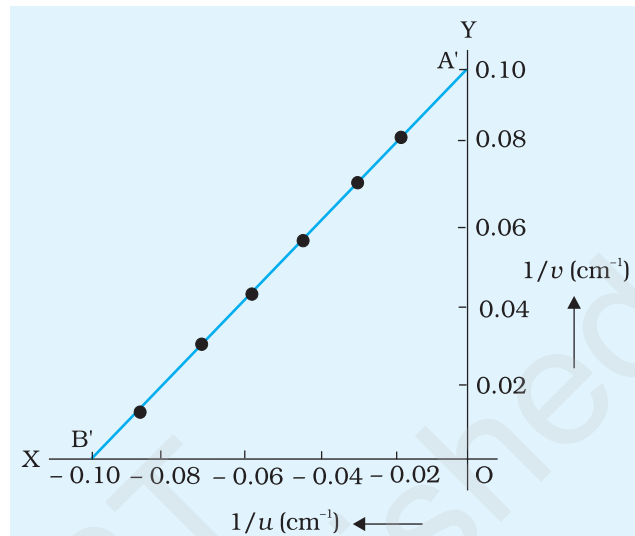


Fig. E 10.5 $1/u$ versus $1/v$ graph for a convex lens (not to scale).

RESULT

The focal length of the given converging thin convex lens:

- (i) from calculations as shown in Observation Table E10.1
 $f \pm \Delta f = \dots \text{cm}$ (here f is mean value of the focal length)
- (ii) from $u - v$ graph $= \dots \text{cm}$, and
- (iii) from $1/u - 1/v$ graph $= \dots \text{cm}$.

PRECAUTIONS

1. The uprights supporting the optical elements should be rigid and mounted vertically.
2. The aperture of the lens should be small otherwise the image formed will not be distinct.
3. Eye should be placed at a distance more than 25 cm from the image needle.
4. An error may arise in the observations if the top of the optical bench is not horizontal and similarly if the tips of pins and optical centre of the lens are not at the same horizontal level.
5. The image and object needles should not be interchanged during the performance of the experiment, as this may cause change in index corrections for object distance and image distance.
6. The tip of the inverted image of the object needle must touch the

tip of the image needle and must not overlap. This should be ensured while removing the parallax.

7. The general instructions to be followed in all optical bench experiments (as given in the description of optical bench) must be taken care of.
8. The corrected values of the distances u and v must be put in the formula for calculating f and then a mean of f should be taken. Calculations for f must not be made using the mean values of u and v .

SOURCES OF ERROR

1. The uprights may not be vertical.
2. Parallax removal may not be perfect.
3. If the knitting needle or index rod for finding index correction is not sharp like a needle, its length may not be accurately found on scale.

DISCUSSION

In plotting $1/v$ versus $1/u$ graph, if scales for the two axes are not same, then the straight line graph may (rather will) not be at 45° to x-axis. This may result in confusions and error in drawing the graph. Keeping the scale same and drawing the best fit graph at 45° to x-axis is the best method. Then, due to inherent errors in measurement $1/f$ on both axes may be a bit too large or a bit too small.

SELF ASSESSMENT

1. Draw the ray diagram for image formation in case of a convex lens for position of object varying from infinity to optical centre.
2. What are the differences between the image formed by a convex lens and a concave lens?
3. How does the focal length of a thick convex lens differ from that of a thin lens?
4. How can you recognise a convex lens, a circular glass slab and a concave lens, without touching them?
5. Where does the centre of curvature of the plane surface of a plano-convex lens lie?
6. Define the principal axis of a plano-convex lens?

7. How does the focal length of a convex lens change if it is dipped in water?
8. What is the relation between focal length and radius of curvature of a plano-convex lens?
9. Can a virtual image produced by a lens be inverted?

SUGGESTED ADDITIONAL EXPERIMENTS/ACTIVITIES

1. Draw a graph by plotting u/v along y-axis and $u + v$ along x-axis. Determine focal length f of the convex lens from the slope.
2. **EFFECT OF MEDIUM ON FOCAL LENGTH**

You have an aquarium, an open window at some distance from it, and a magnifying glass of 50 mm diameter. With your 30 cm scale find the rough focal length of the magnifying glass in air. Then dip it in water by left hand and a white plastic bag (folded with a 5 cm \times 5 cm card in it to make a white screen) by right hand. Focus image of a distant object on the screen by adjusting the position of the screen. Is the rough focal length in water bigger or smaller than that in air? Let a friend measure the focal length in water and find the ratio of the two.
3. **MEASURING LENGTH OF THE FILAMENT OF A CLEAR ELECTRIC LAMP**

You cannot put the scale behind and in contact with the filament to measure its length. Of course you can do this measurement by vernier microscope. But can it be done by a simple convex lens and a scale only? You can also add things like clamp stand etc, as per needs of the experiment. Can you also measure the dark (cool) gap between adjacent segments of the filament of the lamp?

EXPERIMENT 11

AIM

To find the focal length of a convex mirror using a convex lens.

APPARATUS AND MATERIAL REQUIRED

An optical bench with uprights for holding lens, mirror and two needles, two needles (pins), a thin convex lens, a convex mirror, index needle (may be a knitting needle or a pencil sharply pointed at both ends), a metre scale and a spirit level.

PRINCIPLE

Fig. E 11.1 illustrates the formation of image of an object AB by a convex mirror MM' (having a small aperture) in two different situations. The image formed by a convex mirror is virtual and erect. Therefore, its focal length cannot be determined directly. However, it can be determined by introducing a convex lens in between the object and the convex mirror (Fig. E 11.2).

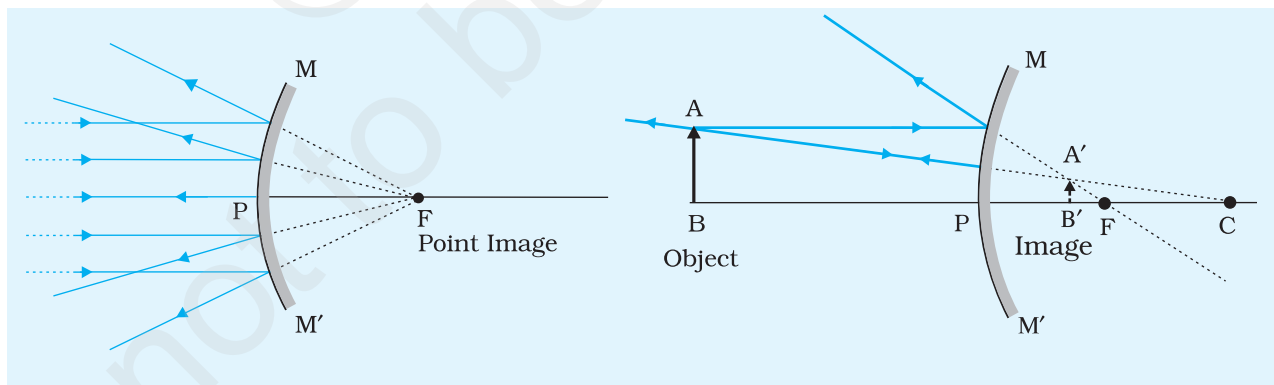


Fig. E 11.1(a) Object is at infinity. A highly diminished and point image is located at the focus behind the convex mirror

Fig. E 11.1(b) Object is in front of the mirror. A diminished virtual image is produced between the pole and focus behind the mirror

An object AB is placed at point P' in front of a thin convex lens such that its real, inverted and magnified image A'B' is formed at position

C on the other side of the lens [Fig. E 11.2(b)]. Now a convex mirror is introduced between the convex lens and point C and so adjusted that the real and inverted image $A'B'$ coincides with the object AB at point P' [Fig. E 11.2 (a)]. This is possible if the light rays starting from the tip of the object, after passing through the lens, fall normally on the reflecting surface of the convex mirror and retrace their path. Any normal ray (perpendicular) to a spherical surface has to be along the radius of that sphere so that point C must be the centre of curvature of the convex mirror. Therefore, the distance PC is the radius of curvature R and half of it would be the focal length of the convex mirror. That is,

$$f = \frac{PC}{2} = \frac{R}{2}$$

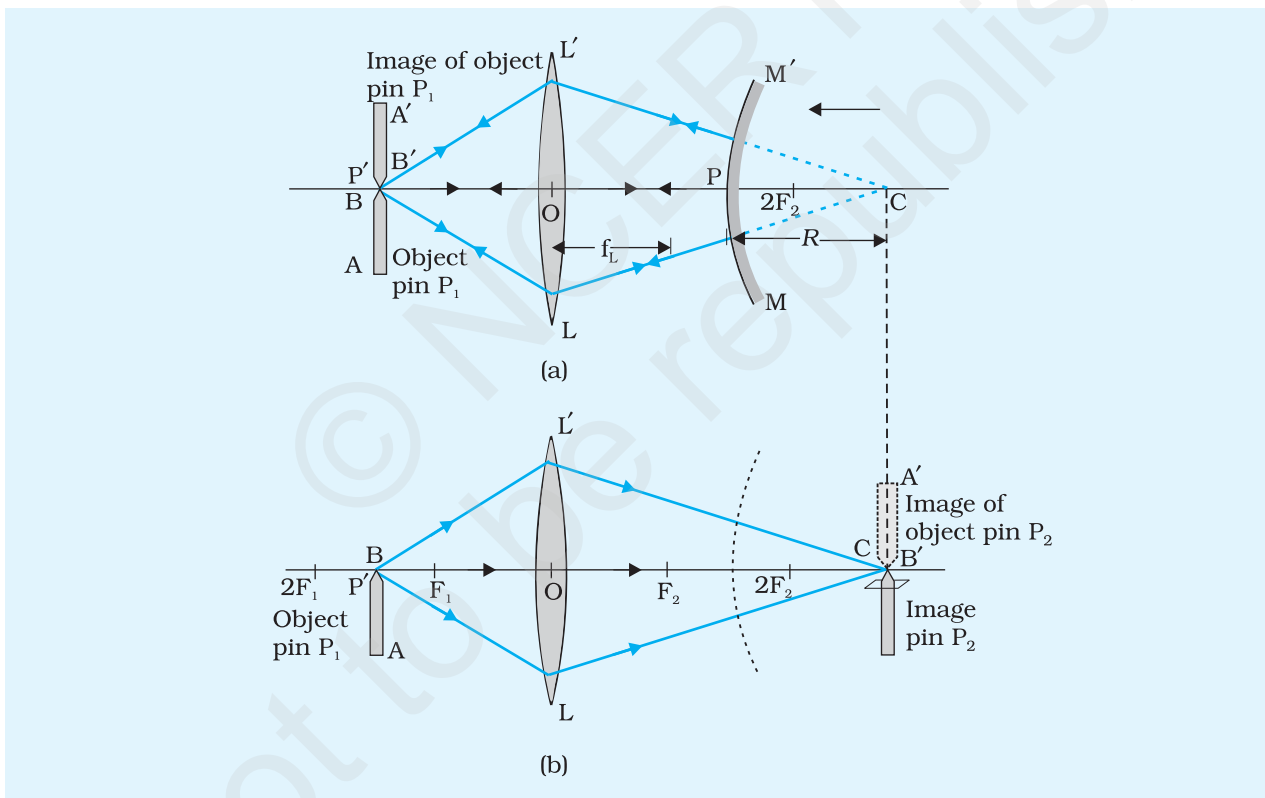


Fig. E 11.2 Image formed by (a) convex mirror and convex lens-image $A'B'$ coincides with the object AB at P' (b) convex lens- image is inverted and magnified

PROCEDURE

1. In case, if the focal length of the given thin convex lens is not known then approximate value of its focal length should be estimated first.

2. Place the optical bench on a rigid table or on a platform. Using the spirit level, make it horizontal with the help of levelling screws provided at the base of the bench.
3. Place the uprights mounted with pin P_1 (object pin), convex lens LL' , and convex mirror MM' on the horizontal optical bench [Fig. E 11.2(a)].
4. Check that the lens, mirror, and pin P_1 are vertically placed on the optical bench. Also verify that the tip of the pin, optical centre O of the convex lens LL' , and pole P' of the convex mirror MM' lie on the same horizontal straight line, parallel to the optical bench.
5. Determine the index correction between upright holding of the convex mirror and image pin respectively, using an index needle.
6. Place the object pin P_1 from the convex lens LL' at a distance slightly greater than the focal length of the lens.
7. Adjust the position of the convex mirror MM' till the light rays reflected back from the mirror pass through the lens and form a real and inverted image coinciding with the object pin P_1 , as shown in Fig. E 11.2 (a). This occurs when the rays starting from the tip of pin P_1 , after passing through the lens strike the mirror normally and are reflected back along their original paths. Remove the parallax between the image and object pins.
8. Read the position of uprights holding the object pin P_1 , convex lens LL' , and convex mirror MM' and record the observations in the observation table.
9. Remove the convex mirror from its upright and fix image pin P_2 on it. Adjust the height of pin such that the tip of it also lies on the principal axis of the lens. That is, the tips of the pins P_1 and P_2 and the optical centre O of the convex lens, all lie on a straight horizontal line parallel to the length of the optical bench.
10. You may put a small piece of paper on image pin P_2 to differentiate it from the object pin P_1 .
11. Using the method of parallax and without changing the position of lens LL' and object pin P_1 , adjust the position of image pin P_2 on the other side of the lens so that it coincides with the real and inverted image of the object pin P_1 formed by the convex lens [Fig. E 11.2(b)]. Note the position of the image pin.
12. Repeat the experiment by changing the separation between the pin P_1 and lens LL' and the mirror MM' . In this manner, take five sets of observations.

OBSERVATIONS

1. Focal length of the convex lens, f (estimated/given) = ... cm
2. Actual length of the index needle, l = ... cm
3. Observed length of the index needle l'
= Position of mirror upright – position of pin upright on the scale
= ... cm
4. Index correction, e = Actual length – observed length ($l - l'$) = ... cm

Table E 11.1: Determination of radius of curvature of convex mirror, R

Sl. No.	Upright position of				Observed $R' = c - d$ (cm)	Corrected R Observed $R' + e$ (cm)	Focal length f (cm)	Δf (cm)
	Object pin P_1 a (cm)	Convex lens LL' b (cm)	Convex mirror MM' c (cm)	Image pin P_2 d (cm)				
1								
2								
...								
5								
					Mean			

CALCULATIONS

Calculate the mean value of radius of curvature of the convex mirror, R , and determine its focal length using the following relation

$$f = \frac{R}{2} = \dots \text{ cm}$$

Error

$$f = \frac{R+l}{2} = \frac{(c-d)+(l-l')}{2}$$

$$\frac{\Delta f}{f} = \frac{\Delta c}{c} + \frac{\Delta d}{d} + \frac{\Delta l}{l} + \frac{\Delta l'}{l'}$$

when Δc , Δd , Δl and $\Delta l'$ are the least counts of the measuring instruments. Maximum of the five values of Δf is to be reported with the result as the experimental error.

RESULT

The focal length of the given convex mirror is $(f \pm \Delta f) \dots \pm \dots$ cm.

Here f is mean value of the focal length.

PRECAUTIONS

1. The uprights supporting the pins, lens and mirror must be rigid and mounted vertically.
2. The apertures of the given convex lens and convex mirror should be small, otherwise the image formed will be distorted.
3. Eye should be placed at a distance of about 25 cm or more from the image pin.
4. Optical bench should be horizontal. The tips of pins, centre of convex lens and pole of the mirror should be at the same horizontal level.

SOURCES OF ERROR

1. The tip of the inverted image of the object pin should just touch the tip of the image pin and must not overlap. This should be ensured while removing the parallax.
2. Personal eye defects may make removal of parallax tedious.
3. The convex mirror should preferably be front-coated. Otherwise multiple reflections may take place.

DISCUSSION

It may not be possible to perform this experiment with just any convex lens. The focal length of the lens used in this experiment should neither be too small nor too large. Why?

SELF ASSESSMENT

1. If focal length of the concave mirror is determined, by using convex lenses of different focal lengths, do you expect any change in the result? If yes, what type of change? If not, why not?

2. How will the result change if a convex lens of different refractive indices were used?
3. If the convex lens selected for the experiment has focal length less than that of the convex mirror, how would this selection limit the experiment?

SUGGESTED ADDITIONAL EXPERIMENTS/ACTIVITIES

1. Repeat the experiment by using convex lenses of different focal lengths. Compare and analyse the results.
2. Repeat the experiment by using convex mirrors of different focal lengths with the help of the same convex lens. Discuss the results.

EXPERIMENT 12

AIM

To find the focal length of a concave lens with the help of a convex lens.

APPARATUS AND MATERIAL REQUIRED

An optical bench with uprights for holding the lenses and two needles, a thin concave lens, a convex lens of focal length (~ 15 cm) smaller than that of the concave lens, index needle (may be a knitting needle), a metre scale and a spirit level.

PRINCIPLE

Figs. E 12.1 (a),(b),(c) and (d) illustrate the formation of image $A'B'$ of an object AB by a concave lens. It is clear that the image formed by a

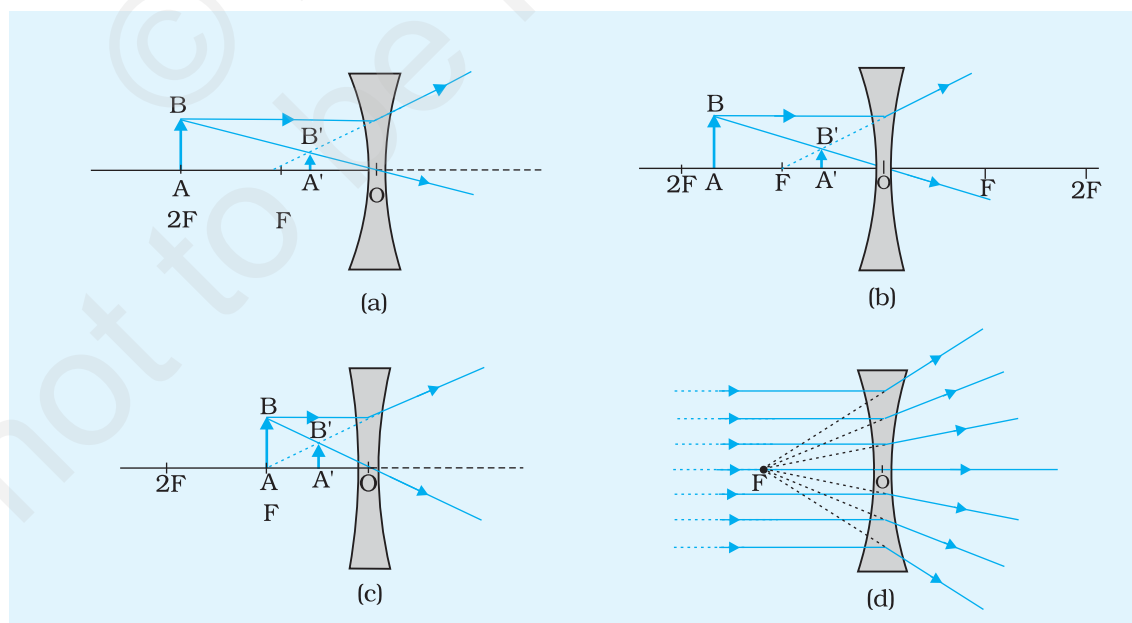


Fig. E 12.1 (a),(b),(c), (d) The images formed by a concave lens for different object positions

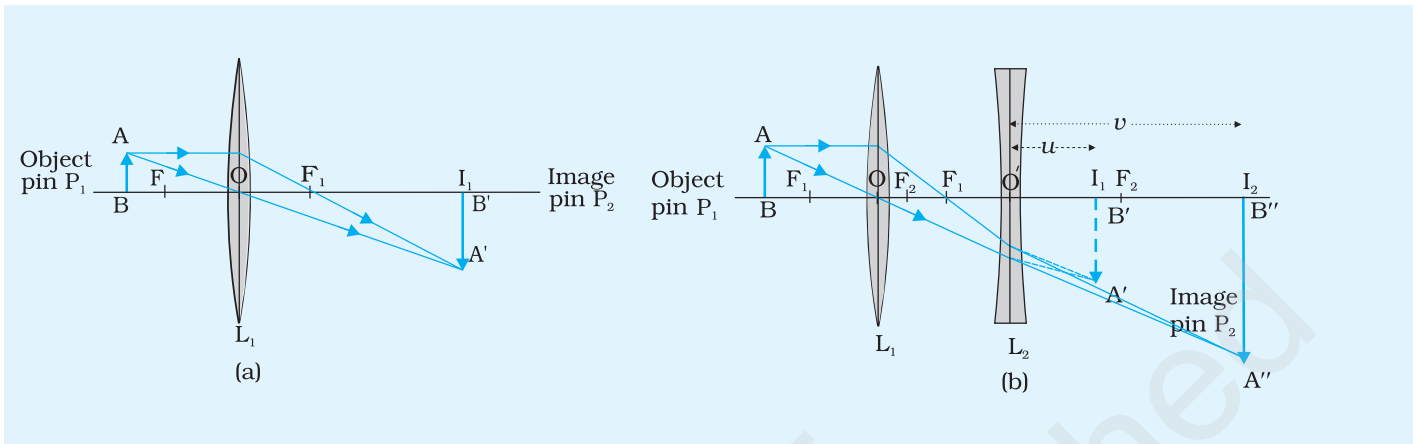


Fig. E 12.2 Formation of image (a) by a convex lens; and (b) by a combination of convex lens and concave lens

concave lens is always virtual and erect in these cases. Therefore, its focal length cannot be determined directly. However, it can be determined indirectly by introducing a convex lens in between the object and the concave lens and producing a real image as illustrated in Fig. E 12.2.

A convex lens L_1 converges the light rays starting from the object AB to form a real and inverted image $A'B'$ at position I_1 [Fig. E 12.2(a)]. If a concave diverging lens L_2 is inserted between the lens L_1 and point I_1 as shown in Fig. E 12.2 (b), for concave lens L_2 image $A'B'$ behaves as virtual object. A real and inverted image $A''B''$ is formed at point I_2 by the diverging lens L_2 . Thus, for the concave lens L_2 the distances $O'I_1$ and $O'I_2$ would be the distances u and v , respectively. It is important to note that the focal length of convex lens L_1 must be smaller than the focal length of the concave lens L_2 . The second image $A''B''$ is formed only when the distance between lens L_2 and first image $A'B'$ is less than the focal length of L_2 .

The focal length of the concave lens L_2 can be calculated from the relation

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \text{ or } f = \frac{uv}{u - v} \quad \text{(E 12.1)}$$

Here for the concave lens both distances u and v are positive and since u will be found to be less than v , f will always be negative.

PROCEDURE

1. In case, if the focal length of the given thin convex lens is not known then rough value of its focal length (f_1) should be estimated first to ensure that its focal length is less than that of the concave lens.

2. Place the optical bench on a rigid platform and using the spirit level, make it horizontal with the help of levelling screws provided at the base of the bench.
3. Place the uprights mounted with pin P_1 (object pin), convex lens L_1 , and another pin P_2 (image pin) on the optical bench. You may put a small piece of paper on image pin P_2 to differentiate it from the image of object pin P_1 [Fig. E 12.2(a)].
4. Check the collinearity of the tip of pin P_1 , optical centre O of convex lens L_1 , and the tip of image pin P_2 along a horizontal straight line which is parallel to the length of the optical bench. In this condition the planes of lens and both the pins would be perpendicular to the axis of the lens.
5. For the determination of the index correction, bring a mounted pin close to the concave lens L_2 . Adjust the index needle (a sharp-edged knitting needle would also serve the purpose) horizontally such that its one end touches one of the curved surfaces of the lens and the other end touches the tip of the pin. Note the positions of the two uprights on the scale provided on the optical bench. The difference of the two would give the **observed length** of the index needle. The **actual length** between the tip of the pin and optical centre O' of the lens L_2 would be length of the index needle (as measured by a scale) plus half of the thickness of the lens at its optical centre. The difference of the two lengths is the index correction.
(If the concave lens is thin at the centre, its thickness at the centre can be ignored).
6. Separate the object pin P_1 from the convex lens by a distance slightly greater than the focal length f_L of the lens.
7. Locate its real and inverted image at point I_1 on the other side of the lens by removing the parallax between the image pin P_2 and image of the object pin P_1 [Fig. E 12.3(a)].
8. Read the positions of the uprights holding the object pin P_1 , convex lens L_1 , and image pin P_2 (i.e. point I_1). Record these observations in Table E 12.1.
9. From now on, do not change the position of the convex lens L_1 and the position of the object pin P_1 . Insert the concave lens L_2 in between the convex lens L_1 and image pin P_2 . Now the image of object pin will shift further from the convex lens L_1 to a point I_2 (say). Adjust the position of the concave lens so that the point I_2 is sufficiently away from the point I_1 .
10. In case the image formed by the combination of convex and concave lenses is not distinctly visible, try to see it on moving the concave lens nearer to the point I_1 and to locate the image by using a pencil

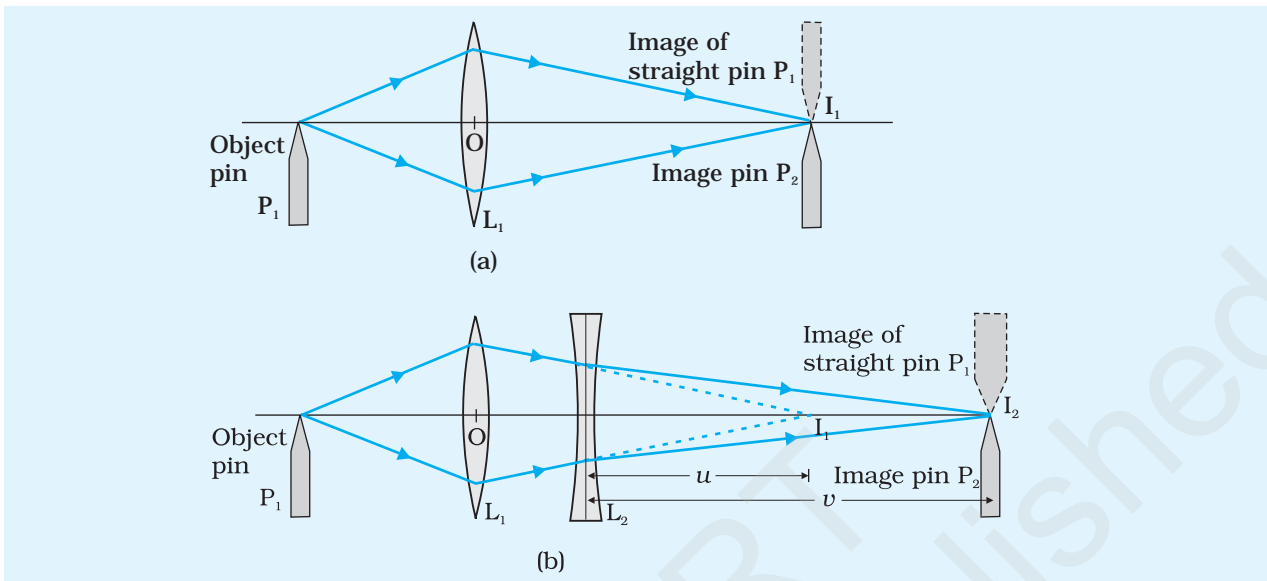


Fig. E 12.3 Focal length of concave lens with the help of convex lens

held in hand, and keeping the image pin P_2 at point I_1 as a guide to decide which way to shift the concave lens L_2 . After having seen the clear image at point I_2 and ensured that it lies within the range of the optical bench, move image pin P_2 to locate the image (or point I_2) more accurately using the method of parallax [Fig. E 12.3(b)]. Since the image forming at I_2 is quite enlarged, it can be blurred.

11. Note the position of uprights holding the concave lens and image pin P_2 , *i.e.*, point I_2 . Note the readings in the Observation Table.
12. Change the position of upright holding the object pin P_1 and repeat the steps 6 to 10. Take five sets of observations.

OBSERVATIONS

1. Focal length of the convex lens, $f_L = \dots$ cm
2. Length of the index needle as measured by the scale, $s = \dots$ cm
3. Thickness of the thin concave lens (given) at its optical centre, $t = \dots$ cm
4. Actual length between the optical centre O of the lens and tip of the pin, $l = s + t/2 = \dots$ cm
5. Observed length of the index needle, l'
 = Distance between the pole of the lens and tip of the pin
 = Position of lens upright - position of pin upright on the scale
 = \dots cm

Table E 12.1: Determination of u , v , and f of concave lens

Sl. No.	Position of										
	Object pin upright P_1 , a (cm)	Convex lens L_1 upright, b (cm)	Image formed by L_1 , point I_1 , c (cm)	Concave lens L_2 upright, d (cm)	Image formed by L_1 and L_2 , point I_2 , g (cm)	Observed u = $c - d$ (cm)	Observed v = $g - d$ (cm)	Corrected u = Observed $u + e$ (cm)	Corrected v = Observed $v + e$ (cm)	$f = uv/(u - v)$ (cm)	Δf (cm)
1											
2											
...											
5											
	Mean										

6. Index correction, $e = l - l' = \dots$ cm

CALCULATIONS

Find the focal length of the concave lens using the formula $f = \frac{uv}{u - v}$

Error

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{\Delta f}{f^2} = \frac{\Delta v}{v^2} + \frac{\Delta u}{u^2}$$

$$\Delta f = f^2 \left[\frac{\Delta v}{v^2} + \frac{\Delta u}{u^2} \right]$$

where Δu , Δv represent least counts of the measuring scale. Values of u , v , f are to be taken from the Observation Table. Maximum of the five values of the error Δf is to be reported with the result as error.

RESULT

The focal length of the given concave lens is $(f \pm \Delta f) = \dots \pm \dots \text{cm}$.

Here f is mean value of the focal length.

PRECAUTIONS

1. The concave lens must be placed near the convex lens. In fact, the second image I_2 is formed only when the distance between concave lens L_2 and first image I_1 (which acts as virtual object for the concave lens) is less than the focal length of the concave lens.
2. Since the image formed at I_2 is quite enlarged, it can be blurred. Therefore, it would be preferable to use a thin and sharp object pin and shine it with light using a lighted electric bulb.
3. The convex lens and the pin P_1 must not be disturbed during the second part of the experiment.
4. A diminished, real and inverted image of the image pin P_2 might also be formed by the light rays reflecting from the concave surface of the lens L_2 . It should not be confused with the bold and bright image formed by the combination of convex and concave lenses.
5. Index correction/ bench correction for u and v should be made.

SOURCES OF ERROR

1. If tip of object pin and optical centre of the lens are not aligned properly (if not brought at the same horizontal level), image tip and image of object pin tip will not touch each other. There may be some gap between the two or there could be overlap between the two. In such situations, there can be error in removing parallax and it will lead to errors in the result.
2. For greater accuracy we should use sharply pointed object pin.

DISCUSSION

1. As concave lens diverges the rays, the image formed by a concave lens alone will not be real and cannot be taken on a screen. To converge these diverging rays to form a real image, convex lens is used.
2. Diverging rays from concave lens can be made to fall normally on a concave mirror to get the real image formed at the point where object is placed. Hence, the focal length of the concave lens can be found by using a concave mirror also.

3. Since the image I_2 is quite enlarged, it can get blurred by chromatic aberration of the two lenses. Thus it is better to put a screen behind object pin P_1 and thus do the entire experiment with one colour of light instead of with white light. For the same reason, pin P_1 should be quite thin and sharp compared to pin P_2 .

SELF ASSESSMENT

1. In this experimental setup the combination of concave lens and convex lens separated by a distance d behaves as a single lens of focal length F . Check the relation $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$ for any one of the observations.
2. Calculate f by interchanging the value of u and v and compare it with the experimentally determined value of f .

SUGGESTED ADDITIONAL EXPERIMENTS/ACTIVITIES

1. Plot a graph of uv against $u-v$ with uv on y-axis and $u-v$ on x-axis. Determine f from the slope of the graph.
2. Repeat the experiment by using concave and convex lenses of different focal lengths, compare and analyse the results.

(E 13.1)

From geometrical considerations we have

$$r + r' = A$$

(E 13.2)

$$\delta = (i - r) + (e - r') = i + e - A$$

At the position of the prism for minimum deviation δ_m , the light ray passes through the prism symmetrically, i.e. parallel to the base so that when

$$\delta = \delta_m, i = e \text{ which implies } r = r'$$

The advantage of putting the prism in minimum deviation position is that the image is brightest in this position.

PROCEDURE

1. Fix a white sheet of paper on a drawing board with the help of cello tape or drawing pins.
2. Draw a straight line XY, using a sharp pencil nearly in the middle and parallel to the length of the paper.
3. Mark points O_1, O_2, O_3, \dots on the straight line XY at suitable distances of about 8 to 10 cm and draw normals $N_1 O_1, N_2 O_2, N_3 O_3, \dots$ on these points (Fig. E 13.2).

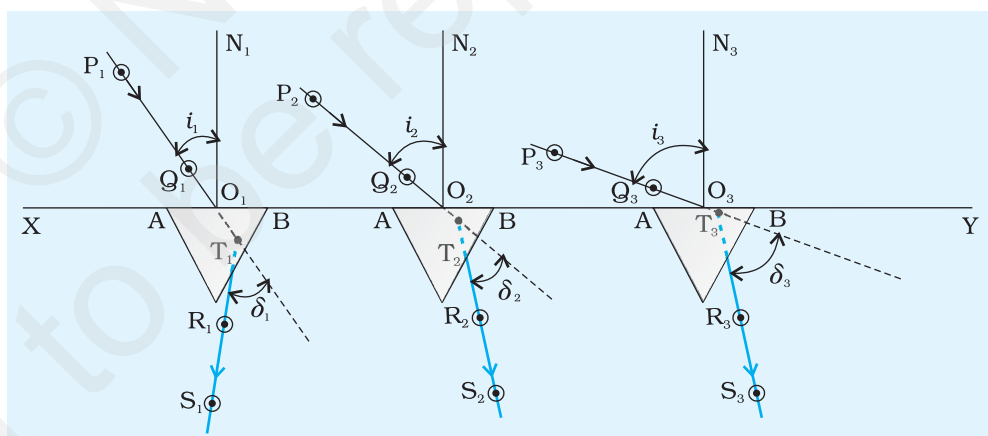


Fig. E 13.2 Refraction of light through a glass prism for various angles of incidence

4. Draw straight lines, $P_1 O_1, P_2 O_2, P_3 O_3, \dots$ corresponding to the incident rays making angles of incidence at $35^\circ, 40^\circ, 45^\circ, 50^\circ, \dots 60^\circ$ respectively with the normals, using a protractor. Write the values of the angles $\angle P_1 O_1 N_1, \angle P_2 O_2 N_2, \angle P_3 O_3 N_3, \dots$ on the white paper sheet (Fig. E 13.2).
5. Place the prism with its refracting face AB on the line XY with point O_1 in the middle of AB as shown in the figure. Draw the boundary of the prism with a sharp pencil.

6. Fix two alpins P_1 and Q_1 with sharp tips vertically about 10 cm apart, on the incident ray line $P_1 Q_1$ such that pin Q_1 is close to point O_1 . Close one eye (say left) and looking through the prism, bring your right eye in line with the images of the pins P_1 and Q_1 . Fix alpins R_1 and S_1 about 10 cm apart vertically on the white paper sheet with their tips in line with the tips of the images of pins P_1 and Q_1 . In this way pins R_1 and S_1 will become collinear, with the images of pins P_1 and Q_1 .
7. Remove the pins R_1 and S_1 and encircle their pin pricks on the white paper sheet with the help of a sharp pencil. Remove the pins P_1 and Q_1 and encircle their pin pricks also.
8. Join the points (or pin pricks) R_1 and S_1 with the help of a sharp pencil and scale, to obtain the emergent ray $R_1 S_1$. Produce it backwards to meet the incident ray $P_1 Q_1$ (produced forward) at T_1 . Draw arrowheads on $P_1 Q_1$ and $R_1 S_1$ to show the direction of the rays.
9. Measure the angle of deviation δ_1 and the angle BAC (angle A) of the prism (Fig. E 13.1) with a protractor and write the values of these angles indicated in the diagram.
10. Repeat steps 5 to 9 for different values of angle of incidence (40° , 45° , 50° ...) and measure the corresponding angles of deviation δ_2 , δ_3 ... with the protractor, and indicate them in the respective diagrams.
11. Record observations in tabular form with proper units and significant figures.

OBSERVATIONS

Least count of the protractor = ...(degree)

Angle of the prism, A = ...(degree)

Table E 13.1: Measuring the angle of incidence, i and angle of deviation δ for a prism

Sl. No.	Angle of incidence, i (degrees)	Angle of deviation, δ (degrees)
1		
2		
3		
--		
10		

Plotting the graph between i and δ for the prism

Take angle of incidence i along x-axis and angle of deviation δ along y-axis, using the observed values from Table E 13.1. Choose suitable scales on these axes and plot a graph between i and δ . Take care that you draw a free hand smooth curve passing practically through all the plotted points on the graph (Fig. E 13.3).

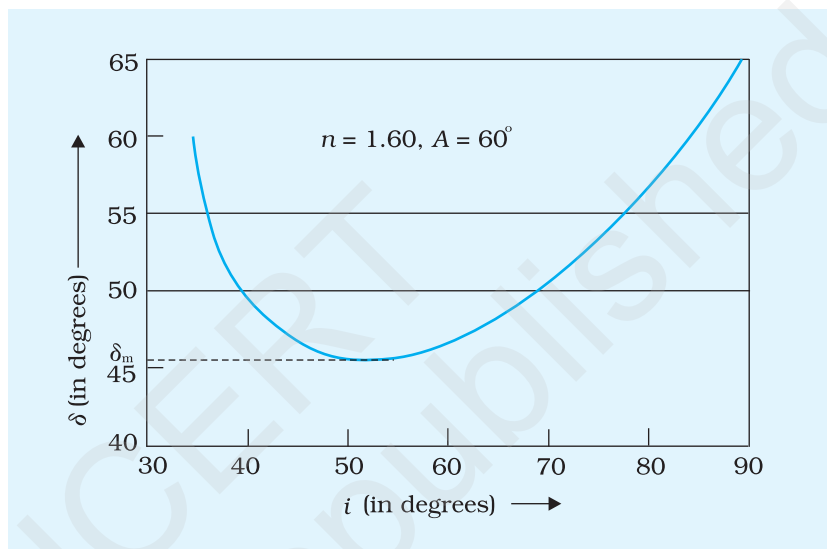


Fig. E 13.3 Graph between angle of incidence and angle of deviation

CALCULATIONS

Draw tangent on the lowest point of the graph parallel to x-axis, read the angle of minimum deviation δ_m on the y-axis of the graph. Express the result with proper significant figures.

RESULT

Angle of minimum deviation, $\delta_m = \dots \pm \dots$ degree

PRECAUTIONS

1. Alpins should be fixed vertically to the plane of paper.
2. Distance PQ and RS should be about 10 cm in order to locate incident and emergent rays with greater accuracy.
3. Same angle of prism should be used for all observations.
4. Position of the prism should not be disturbed for a given set of observations.

SOURCES OF ERROR

1. If the three angles of refraction between adjacent pairs of faces are not equal, then $A + \delta \neq i + e$.
2. There may be an error in measuring the values of the angles.

DISCUSSION

1. It is suggested that the value of angle of incidence be taken more than 35° . This is required for angles less than 35° as there is a possibility of total internal reflection inside the prism.
2. You must check your readings by applying the formula $i + e = A + \delta$.
3. The $i - \delta$ curve that is obtained in this experiment is a non-linear curve. In such situations, more readings should be taken in the minimum deviation region to be able to obtain the value of angle of minimum deviation accurately. For example, if δ readings are taken initially at 35° , 40° , 45° and 50° and if the $i - \delta$ data points are situated as shown in Fig. E 13.3 then a few more readings need to be taken for values of i in the range 45° to 55° say, at a difference of 1° or 2° .

Taking more readings in this region will help in drawing a smooth curve. This will enable you to locate the position of the lowest point on the graph more accurately.

4. In the condition of minimum deviation, the refracted ray inside the prism becomes parallel to its base so as to satisfy the condition $r = r'$.
5. The graph does not show a sharp minimum. We have same deviation for a range of angle of incidence near minimum deviation. Therefore extra care should be taken in drawing tangential line to the $i - \delta$ graph at minimum deviation.

SELF ASSESSMENT

1. Interpret the graph between i and δ .
2. If the experiment is performed with angle of emergence taken as angle of incidence, will there be any change in the δ values? If yes, why? If not, why not?
3. What will happen if you go on decreasing the angle of incidence? If you think there is a minimum, try to find its expression theoretically. What happens when i is less than the minimum angle of incidence?

SUGGESTED ADDITIONAL EXPERIMENTS/ACTIVITIES

1. Use the following equation to calculate refractive index of the material of the prism from the measured value of the angle A and δ_m determined from the graph.

$$n = \frac{\sin i}{\sin r} = \frac{\sin [(A + \delta_m)/2]}{\sin(A/2)}$$

2. Measure the angle of emergence e corresponding to the value of each angle of incidence i and angle of deviation δ you have observed. Compute the values of $(i + e)$ and $(A + \delta)$ and see how they compare.
3. Draw parallel horizontal lines cutting the $i - \delta$ curve, you have drawn, at various values of i and e . Find out the mid points of these horizontal lines and join these mid points. What is the shape of the curve so obtained? If you find that this shape resembles that of a straight line, find its (i) slope (ii) y-intercept and (iii) x-intercept.
4. Determine the refractive index of different liquids using a hollow prism by plotting $i - \delta$ graph.
5. Measure r and r' and e from the figures you have drawn. Find the refractive index of the material of the prism from the values of i and r and e and r' .

DATA SECTION

TABLE D_s1.1: Some important constants

Name	Symbol	Value
Speed of light in vacuum	c	$2.9979 \times 10^8 \text{ m s}^{-1}$
Charge of electron	e	$1.602 \times 10^{-19} \text{ C}$
Gravitational constant	G	$6.673 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$
Planck constant	h	$6.626 \times 10^{-34} \text{ J s}$
Boltzmann constant	k	$1.381 \times 10^{-23} \text{ J K}^{-1}$
Avogadro number	N_A	$6.022 \times 10^{23} \text{ mol}^{-1}$
Universal gas constant	R	$8.314 \text{ J mol}^{-1} \text{ K}^{-1}$
Mass of electron	m_e	$9.110 \times 10^{-31} \text{ kg}$
Mass of neutron	m_n	$1.675 \times 10^{-27} \text{ kg}$
Mass of proton	m_p	$1.673 \times 10^{-27} \text{ kg}$
Electron-charge to mass ratio	e/m_e	$1.759 \times 10^{11} \text{ C/kg}$
Faraday constant	F	$9.648 \times 10^4 \text{ C/mol}$
Rydberg constant	R	$1.097 \times 10^7 \text{ m}^{-1}$
Bohr radius	a_0	$5.292 \times 10^{-11} \text{ m}$
Stefan-Boltzmann constant	σ	$5.670 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$
Wien's constant	b	$2.898 \times 10^{-3} \text{ m K}$
Permittivity of free space	ϵ_0	$8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$
	$1/4\pi\epsilon_0$	$8.987 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$
Permeability of free space	μ_0	$4\pi \times 10^{-7} \text{ T m A}^{-1}$ $\cong 1.257 \times 10^{-6} \text{ Wb A}^{-1}\text{m}^{-1}$

TABLE D_s 1.2: Other useful constants

Name	Symbol	Value
Mechanical equivalent of heat	J	4.186 J cal^{-1}
Standard atmospheric pressure	1 atm	$1.013 \times 10^5 \text{ Pa}$
Absolute zero	0 K	$-273.15 \text{ }^\circ\text{C}$
Electron volt	1 eV	$1.602 \times 10^{-19} \text{ J}$
Unified Atomic mass unit	1 u	$1.661 \times 10^{-27} \text{ kg}$
Electron rest energy	mc^2	0.511 MeV
Energy equivalent of 1 u	1 uc^2	931.5 MeV
Volume of ideal gas (0 °C and 1 atm)	V	22.4 L mol^{-1}
Acceleration due to gravity (sea level, at equator)	g	9.78049 m s^{-2}

TABLE D_s 2: E. M.F. of cells

Cell	E.M.F. (V)	Cell	E.M.F. (V)
Daniell	1.08 – 1.09	Cadmium at 20°C	1.018 54
Grove	1.8 – 1.9	Lead accumulator	1.9 – 2.2
Lechlanche	1.45	Edison cell	1.45
Voltaic	1.01	Clarke	1.43
Bunsen	1.95	Ni-Fe	1.20

TABLE D_s 3: Refractive index of substances : For sodium light $\lambda = 5896 \text{ } 10^{-10} \text{ m}$

Solid	Refractive index	Liquid	Refractive Index
Diamond	2.417	Canada balsam	1.53
Glass (crown)	1.48 – 1.61	Water	1.333
Glass (flint)	1.53 – 1.96	Alcohol (ethyl)	1.362
Glass(soda)	1.50	Aniline	1.595
		Benzene	1.501
Ice	1.31	Cedar oil	1.516
Mica	1.56 – 1.60	Chloroform	1.450
Rock-salt	1.54	Ether	1.350
		Glycerine	1.47
Quartz (O - Ray)	1.5443	Olive oil	1.46
Quartz (E - Ray)	1.5534	Paraffin oil	1.44
Quartz (fused)	1.458	Kerosene oil	1.39
		Turpentine oil	1.44

TABLE D_s 4: Wavelength of spectral lines : (in Å, $1\text{Å} = 10^{-10} \text{ m}$)

[The visible spectrum colours are indicated – *r, o, y, g, b, i, v*]

Hydrogen	Helium	Mercury	Neon	Sodium
3970 <i>v</i>	3889 <i>v</i>	4047 <i>v</i>	5765 <i>y</i>	(D ₂) 5890 <i>o</i>
4102 <i>v</i>	4026 <i>v</i>	4078 <i>v</i>	5853 <i>y</i>	(D ₁) 5896 <i>o</i>
4340 <i>b</i>	4471 <i>b</i>	4358 <i>v</i>	5882 <i>o</i>	
4861 <i>gb</i>	5876 <i>y</i>	4916 <i>b, g</i>	6597 <i>r</i>	
6563 <i>r</i>	6678 <i>r</i>	4960 <i>g</i>	7245 <i>r</i>	
	7065 <i>r</i>	5461 <i>g</i>		
		5770 <i>y</i>		
		5791 <i>y</i>		
		6152 <i>o</i>		
		6322 <i>o</i>		

TABLE D₅: Electromagnetic spectrum (wavelength)

Range		Wavelength	
Wireless waves		5 m and above	
Infra-red	3.0 10^{-4} m	to	7.5 10^{-7} m
Visible Red	7.5 10^{-7} m	to	6.5 10^{-7} m
Visible Orange	6.5 10^{-7} m	to	5.9 10^{-7} m
Visible Yellow	5.9 10^{-7} m	to	5.3 10^{-7} m
Visible Green	5.3 10^{-7} m	to	4.9 10^{-7} m
Visible Blue	4.9 10^{-7} m	to	4.2 10^{-7} m
Visible Indigo	4.2 10^{-7} m	to	3.9 10^{-7} m
Ultra Violet	3.9 10^{-7} m	to	1.8 10^{-7} m
Soft X-Rays	2.0 10^{-7} m	to	1.0 10^{-7} m
Hard X-Rays	1.0 10^{-10} m	to	1.0 10^{-11} m
Gamma Rays	5.0 10^{-11} m	to	5.0 10^{-12} m

TABLE D₆: Standard Wire Gauges (S.W.G.) and resistances for a metre length of constantan and manganin

S.W.G. No.	Diameter (mm)	Resistance (Ω)		
		Copper	Constantan (60% Cu, 40% Ni)	Manganin (84% Cu, 4% Ni, 12% Mn)
10	3.25	0.0021	0.057	0.051
12	2.64	0.0032	0.086	0.077
14	2.03	0.0054	0.146	0.131
16	1.63	0.0083	0.228	0.204
18	1.22	0.0148	0.405	0.361
20	0.914	0.0260	0.722	0.645
22	0.711	0.0435	1.20	1.07
24	0.559	0.070	1.93	1.73
26	0.457	0.105	2.89	2.58
28	0.374	0.155	4.27	3.82
30	0.315	0.222	6.08	5.45
32	0.274	0.293	8.02	7.18
34	0.234	0.404	11.1	9.9
36	0.193	0.590	16.2	14.5
38	0.152	0.950	26.0	23.2
40	0.122	1.48	40.6	36.3
42	0.102	2.10	58.5	53.4
44	0.081	3.30	91.4	81.7
46	0.061	5.90	162.5	145.5

TABLE D_s 7: Dielectric Constants of Common Materials

Material	Temperature (°C)	Frequency (Hz)	Dielectric Constant
Amber	20	10 ⁶	2.8
Amber	20	3 × 10 ⁹	2.6
Soda glass	20	10 ⁶	7.5
Fused quartz	20	10 ³ to 10 ⁸	3.8
Liquid paraffin (Medical Grade)	20	10 ³	2.2
Transformer oil (Class B)	20	10 ³	2.2
Marble	20	10 ⁶	8
Sand (dry)	20	10 ⁶	3
Sandstone	20	10 ⁶	10
Paper (Oil impregnated condenser tissue)	20	10 ³	2.3
Mica	20	10 ³ to 10 ⁸	5.4 to 7
Epoxy resin (e.g. Araldite)	20	10 ⁶	3.3
Cellulose Acetate	20	10 ⁶	3.5
Vinyl Acetate (Plasticised)	20	10 ⁶	4
Vinyl Chloride (P.V.C.)	20	10 ⁶	4
Ebonite (Pure)	20	10 ⁶	3
Rubber (Vulcanised soft)	20	10 ⁶	3.2
Rubber, Synthetic	20	10 ⁶	2.5
Paraffin wax	20	10 ⁶	2.2
Sulphur	20	3 × 10 ⁹	3.4
Walnut wood (dry)	20	10 ⁷	2.0
Walnut (17% moisture)	20	10 ⁷	5
Vacuum	NA	any	1.00000
Air	20	Upto 3 × 10 ⁹	1.00054
Porcelain	20	10 ⁶	5.5
Barium titanate	20	10 ⁶	1200
Rutile group	20	10 ⁶ to 10 ⁹	40 to 80
Water	20	10 ⁹	80
Water	20	10 ¹⁰	64

TABLE D_s 8: Typical objects with electrostatic charge

Object	C	V	Q	Energy $E = \frac{1}{2} C V^2$
1. Balloon of 20cm diameter rubbed all round by nylon cloth	11 pF	200 V	2.2 nC	0.22 μ J*
2. Metal sphere on insulated stand rubbed by silk (dia 9 cm)	5 pF	500 V	2.5 nC	0.62 μ J*
3. Metal sphere charged by a school type Van-de-Graaff generator	5 pF	0.25 MV	1.25 μ C	0.16 J
4. Boy on an insulated stool repeatedly charged by electrophorus	50 pF	3000 V	150 nC	225 μ J*
5. 30 cm 30 cm improvised capacitor with wax soaked tissue paper dielectric (K = 2.7, A = 700 cm and d = 0.4 mm) and charged by a 9 V battery.	4 nF	9 V	36 nC	162 nJ*
6. -do- plates separated to 2 cm apart.	80 pF	450 V	36 nC	8.1 μ J*
7. Flash gun capacitors (professional type)	500 pF	400 V	0.2 nC	40 J
8. Average lighting between earth and cloud ($h = 1$ to 5 km)		10 ⁸ V to 10 ⁹ V	20 C	10 ⁹ to 10 ¹⁰ J
9. (a) Earth and ionised air of high conductivity at top of stratosphere ($h = 50$ km)	-	0.4 MV	5.7 10 ⁵ C	10 ¹¹ J
(b) Ionisation current between earth and ionosphere in fair weather.	-	0.4 MV	1800 C/s	7 10 ⁸ J/s
10. Charged plate of good electrophorous (diameter 20 to 30 cm) in dry weather, after lifting it up.	10 pF	3000 V	30 nC	45 μ J*

*These bodies in your school laboratory are not dangerous to touch.

TABLE D₉: Electrical resistivities of typical metals and alloys
Resistivity (10^{-8} ohm metre)

Material	0°C	100°C	300°C	700°C	Temperature coefficient at 0°C (over range 0°C-100°C (10^{-4}))
Aluminium	2.45	3.55	5.9	24.7	45
Chromium	12.7	16.1	25.2	47.2	
Copper	1.56	2.24	3.6	6.7	43
Iron	8.9	14.7	31.5	85.5	65
Lead	19.0	27.0	50	107.6	42
Gold	2.04	2.84			40
Mercury	94.0766*	103.5	128		10
Nickel	6.14	10.33	22.5		68
Platinum	9.81	13.65	21.0	34.3	39.2
Platinum- Rhodium (87/13)	19.0	22.0			15.6
Platinum- Rhodium (90/10)	18.7	21.8			16.6
Platinum- Iridium (90/10)	24.8	28.0			13
Silver	1.51	2.13	3.42	6.5	41
Tin	11.5(20°C)	15.8	50	60	46
Tungsten	4.9	7.3	12.4	24	48
Zinc	5.5	7.8	13.0	37(500°C)	42

Mercury at 0°C is used as a secondary standard to realise the standard unit of resistance, ohm.

TABLE D_s 10: Electrical resistivities of common insulators and semiconductors

Substance	Resistivity (ohm metre)	Substance	(Resistivity (ohm metre)
Diamond	10^{10} to 10^{11}	Carbon 0°C	3.5×10^{-5}
Ebonite	10^{14}	Carbon 500°C	2.7×10^{-5}
Glass (Soda lime)	5×10^9	Carbon 1000°C	2.1×10^{-5}
Glass (Pyrex)	10^{12}	Carbon 2000°C	1.1×10^{-5}
Glass (conducting)	5×10^6	Carbon 2500°C	0.9×10^{-5}
Mica	10^{11} to 10^{15}	Germanium 0°C	0.46
Paper (dry)	10^{10}	Silicon 0°C	2300
Paraffin wax	10^{14}		
Porcelain	10^{10} to 10^{13}		
Sulphur (rhombic)	2×10^{21}		

TABLE D_s : 11 Data for intrinsic and extrinsic semiconductors

Material	Energy Gap (eV)	No. density per m^3 * electron-hole pairs at 300 K	Mobility Electrons ($m^2 V^{-1} s^{-1}$)	Mobility holes ($m^2 V^{-1} s^{-1}$)	Conductivity ($S m^{-1}$)	Density ($kg m^{-3}$)
Semiconductors						
Germanium	0.76	6×10^{19}	0.39	0.19	2.18	5320
Silicon	1.12	7×10^{15}	0.135	0.048	4.4×10^{-4}	2300
Doped Si with P with Insulators	0.045	2.5×10^{21}	0.135	...	2.5×10	2300
Diamond	6 to 12	$\approx 10^7$				

*Values are approximate, they give an idea about order of magnitude only.

TABLE D_s 12: Alloys of high resistance

Alloy	Resistivity 20 °C (10⁻⁸ ohm metre)	Temperature coefficient in range 0-100°C (10⁻⁴)	Max. Operating temp- erature (°C)
Constantan (58.8% Cu, 40% Ni, 1.2% Mn)	44 to 52	- 0.4 to + 0.1	500
German Silver (65% Cu, 20% Zn, 15% Ni)	28 to 35	+ 0.4	150 to 200
Manganin (85% Cu, 12% Mn, 3% Ni)	42 to 48	0.3	100
Nickeline (54% Cu, 20% Zn, 26% Ni)	39 to 45	0.2	150 - 200
Nichrome (67.5% Ni, 15% Cr, 16% Fe, 1.5% Mn)	100 to 110	2.0	1000

TABLE D_s 13: Transition temperature to the superconducting state

Substance	Transition temperature (K)	Substance	Transition temperature (K)
Metals:		Compounds:	
Cadmium	0.6	NiBi	4.2
Zinc	0.8	PbSe	5.0
Aluminium	1.2	NbB	6.0
Uranium	1.3	Nb ₂ C	9.2
Tin	3.7	nBC	10.1 to 10.5
Mercury	4.7	nBN	15 to 16
Lead	7.3	Nb ₃ Sn	18
Niobium	9.2	YBa ₂ Cu ₃ O ₇	90

LOGARITHMS

TABLE I

N	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	5 9 13	4 8 12	17 21 26	16 20 24	30 34 38	28 32 36			
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4 8 12	4 7 11	16 20 23	15 18 22	27 31 35	26 29 33			
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3 7 11	3 7 10	14 18 21	14 17 20	25 28 32	24 27 31			
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3 6 10	3 7 10	13 16 19	13 16 19	23 26 29	22 25 29			
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3 6 9	3 6 9	12 15 19	12 14 17	22 25 28	20 23 26			
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3 6 9	3 6 8	11 14 17	11 14 17	20 23 26	19 22 25			
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3 6 8	3 5 8	11 14 16	10 13 16	19 22 24	18 21 23			
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	3 5 8	3 5 8	10 13 15	10 12 15	18 20 23	17 20 22			
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2 5 7	2 4 7	9 12 14	9 11 14	17 19 21	16 18 21			
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2 4 7	2 4 6	9 11 13	8 11 13	16 18 20	15 17 19			
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2 4 6	2 4 6	8 11 13	8 10 12	15 17 19	14 16 18			
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2 4 6	2 4 6	8 10 12	8 10 12	14 16 18	13 15 17			
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2 4 6	2 4 6	7 9 11	7 9 11	13 15 17	12 14 16			
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2 4 6	2 4 5	7 9 11	7 9 10	12 14 16	11 13 15			
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2 4 5	2 3 5	7 9 11	7 9 10	12 14 15	11 13 14			
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2 3 5	2 3 5	6 8 9	6 8 9	11 13 14	10 12 13			
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2 3 5	2 3 5	6 8 9	6 8 9	11 13 14	10 12 13			
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2 3 5	2 3 5	5 6 8	5 6 8	10 12 13	9 11 12			
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2 3 5	1 3 4	6 7 9	6 7 9	10 12 13	9 11 12			
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1 3 4	1 3 4	6 7 9	6 7 9	10 11 13	9 11 12			
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1 3 4	1 3 4	6 7 9	6 7 8	10 11 12	9 11 12			
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1 3 4	1 3 4	5 7 8	5 7 8	9 11 12	8 9 10			
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1 3 4	1 3 4	5 6 8	5 6 8	9 10 12	8 9 10			
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1 3 4	1 3 4	5 6 8	5 6 8	9 10 11	8 9 10			
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1 3 4	1 3 4	5 6 8	5 6 8	9 10 11	8 9 10			
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1 2 4	1 2 4	5 6 7	5 6 7	8 10 11	7 8 9			
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1 2 4	1 2 3	5 6 7	5 6 7	8 9 10	7 8 9			
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1 2 3	1 2 3	5 6 7	5 6 7	8 9 10	7 8 9			
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1 2 3	1 2 3	5 6 7	5 6 7	8 9 10	7 8 9			
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1 2 3	1 2 3	4 5 7	4 5 7	8 9 10	7 8 9			
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1 2 3	1 2 3	4 5 6	4 5 6	7 8 9	6 7 8			
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1 2 3	1 2 3	4 5 6	4 5 6	7 8 9	6 7 8			
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1 2 3	1 2 3	4 5 6	4 5 6	7 8 9	6 7 8			
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1 2 3	1 2 3	4 5 6	4 5 6	7 8 9	6 7 8			
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1 2 3	1 2 3	4 5 6	4 5 6	7 8 9	6 7 8			
45	6532	6542	6551	6561	6471	6580	6590	6599	6609	6618	1 2 3	1 2 3	4 5 6	4 5 6	7 8 9	6 7 8			
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1 2 3	1 2 3	4 5 6	4 5 6	7 7 8	6 7 8			
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1 2 3	1 2 3	4 5 5	4 5 5	6 7 8	5 6 7			
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1 2 3	1 2 3	4 4 5	4 4 5	6 7 8	5 6 7			
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1 2 3	1 2 3	4 4 5	4 4 5	6 7 8	5 6 7			

LOGARITHMS

TABLE 1 (Continued)

N	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9992	9996	0	1	1	2	2	3	3	4	4

ANTILOGARITHMS

TABLE II

N	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
.00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
.01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
.02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
.03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
.04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
.05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
.06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
.07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
.08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
.09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
.10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
.11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
.12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
.13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
.14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
.15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
.16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
.17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
.18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
.19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
.20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
.21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
.22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
.23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
.24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
.25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
.26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
.27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
.28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
.29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
.30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
.31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
.32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
.33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
.34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
.35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
.36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
.37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
.38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
.39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
.40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
.41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
.42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
.43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
.44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
.45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
.46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
.47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
.48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	3	4	5	6	6
.49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	3	4	5	6	6

ANTILOGARITHMS

TABLE II (Continued)

N	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
.50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
.51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
.52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
.53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
.54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
.55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
.56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
.57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
.58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
.59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
.60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
.61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
.62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
.63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
.64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
.65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
.66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
.67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
.68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
.69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
.70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
.71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
.72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
.73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
.74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
.75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
.76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
.77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
.78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
.79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
.80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
.81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
.82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
.83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
.84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
.85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
.86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
.87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
.88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
.89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
.90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
.91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
.92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
.93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
.94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
.95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
.96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
.97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
.98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
.99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

NATURAL SINES

TABLE I

	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'	Mean				
	0°.0	0°.1	0°.2	0°.3	0°.4	0°.5	0°.6	0°.7	0°.8	0°.9	Differences				
											1'	2'	3'	4'	5'
0	.0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	.0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	.0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	3	6	9	12	15
3	.0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	3	6	9	12	15
4	.0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	3	6	9	12	15
5	.0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	3	6	9	12	14
6	.1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	3	6	9	12	14
7	.1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	3	6	9	12	14
8	.1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	3	6	9	12	14
9	.1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	3	6	9	12	14
10	.1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	3	6	9	12	14
11	.1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	3	6	9	11	14
12	.2079	2096	2113	2130	2147	2164	2181	2198	2215	2232	3	6	9	11	14
13	.2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	3	6	8	11	14
14	.2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	3	6	8	11	14
15	.2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	3	6	8	11	14
16	.2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	3	6	8	11	14
17	.2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	3	6	8	11	14
18	.3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	3	6	8	11	14
19	.3256	3272	3289	3305	3322	3338	3355	3371	3387	3404	3	5	8	11	14
20	.3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	3	5	8	11	14
21	.3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	3	5	8	11	14
22	.3746	3762	3778	3795	3811	3827	3843	3859	3875	3891	3	5	8	11	14
23	.3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	3	5	8	11	14
24	.4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	3	5	8	11	13
25	.4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	3	5	8	11	13
26	.4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	3	5	8	10	13
27	.4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	3	5	8	10	13
28	.4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	3	5	8	10	13
29	.4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	3	5	8	10	13
30	.5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	3	5	8	10	13
31	.5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	2	5	7	10	12
32	.5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	2	5	7	10	12
33	.5446	5461	5476	5490	5505	5519	5534	5548	5563	5577	2	5	7	10	12
34	.5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	2	5	7	10	12
35	.5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	2	5	7	10	12
36	.5878	5892	5906	5920	5934	5948	5962	5976	5990	6004	2	5	7	9	12
37	.6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	2	5	7	9	12
38	.6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	2	5	7	9	11
39	.6293	6307	6320	6334	6347	6361	6374	6388	6401	6414	2	4	7	9	11
40	.6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	2	4	7	9	11
41	.6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	2	4	7	9	11
42	.6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	2	4	6	9	11
43	.6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	2	4	6	8	11
44	.6947	6959	6972	6984	6997	7009	7022	7034	7046	7059	2	4	6	8	10

NATURAL SINES

TABLE I (Continued)

	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'	Mean				
	0°.0	0°.1	0°.2	0°.3	0°.4	0°.5	0°.6	0°.7	0°.8	0°.9	Differences				
											1'	2'	3'	4'	5'
45	.7071	7083	7096	7108	7120	7133	7145	7157	7169	7181	2	4	6	8	10
46	.7193	7206	7218	7230	7242	7254	7266	7278	7290	7302	2	4	6	8	10
47	.7314	7325	7337	7349	7361	7373	7385	7396	7408	7420	2	4	6	8	10
48	.7431	7443	7455	7466	7478	7490	7501	7513	7524	7536	2	4	6	8	10
49	.7547	7558	7570	7581	7593	7604	7615	7627	7638	7649	2	4	6	8	9
50	.7660	7672	7683	7694	7705	7716	7727	7738	7749	7760	2	4	6	7	9
51	.7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	2	4	5	7	9
52	.7880	7891	7902	7912	7923	7934	7944	7955	7965	7976	2	4	5	7	9
53	.7986	7997	8007	8018	8028	8039	8049	8059	8070	8080	2	3	5	7	9
54	.8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	2	3	5	7	8
55	.8192	8202	8211	8221	8231	8241	8251	8261	8271	8281	2	3	5	7	8
56	.8290	8300	8310	8320	8329	8339	8348	8358	8368	8377	2	3	5	6	8
57	.8387	8396	8406	8415	8425	8434	8443	8453	8462	8471	2	3	5	6	8
58	.8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	2	3	5	6	8
59	.8572	8581	8590	8599	8607	8616	8625	8634	8643	8652	1	3	4	6	7
60	.8660	8669	8678	8686	8695	8704	8712	8721	8729	8738	1	3	4	6	7
61	.8746	8755	8763	8771	8780	8788	8796	8805	8813	8821	1	3	4	6	7
62	.8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	1	3	4	5	7
63	.8910	8918	8926	8934	8942	8949	8957	8965	8973	8980	1	3	4	5	6
64	.8988	8996	9003	9011	9018	9026	9033	9041	9048	9056	1	3	4	5	6
65	.9063	9070	9078	9085	9092	9100	9107	9114	9121	9128	1	2	4	5	6
66	.9135	9143	9150	9157	9164	9171	9178	9184	9191	9198	1	2	3	5	6
67	.9205	9212	9219	9225	9232	9239	9245	9252	9259	9265	1	2	3	4	6
68	.9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	1	2	3	4	5
69	.9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	1	2	3	4	5
70	.9397	9403	9409	9415	9421	9426	9432	9438	9444	9449	1	2	3	4	5
71	.9455	9461	9466	9472	9478	9483	9489	9494	9500	9505	1	2	3	4	5
72	.9511	9516	9521	9527	9532	9537	9542	9548	9553	9558	1	2	3	3	4
73	.9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	1	2	3	3	4
74	.9613	9617	9622	9627	9632	9636	9641	9646	9650	9655	1	2	2	3	4
75	.9659	9664	9668	9673	9677	9681	9686	9690	9694	9699	1	1	2	3	4
76	.9703	9707	9711	9715	9720	9724	9728	9732	9736	9740	1	1	2	3	3
77	.9744	9748	9751	9755	9759	9763	9767	9770	9774	9778	1	1	2	3	3
78	.9781	9785	9789	9792	9796	9799	9803	9806	9810	9813	1	1	2	2	3
79	.9816	9820	9823	9826	9829	9833	9836	9839	9842	9845	1	1	2	2	3
80	.9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	0	1	1	2	2
81	.9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	0	1	1	2	2
82	.9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	0	1	1	2	2
83	.9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	0	1	1	1	2
84	.9945	9947	9949	9951	9952	9954	9956	9957	9959	9960	0	1	1	1	2
85	.9962	9963	9965	9966	9968	9969	9971	9972	9973	9974	0	0	1	1	1
86	.9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	0	0	1	1	1
87	.9986	9987	9988	9989	9990	9990	9991	9992	9993	9993	0	0	0	1	1
88	.9994	9995	9995	9996	9996	9997	9997	9997	9998	9998	0	0	0	0	0
89	.9998	9999	9999	9999	9999	1.000	1.000	1.000	1.000	1.000	0	0	0	0	0
90	1.000														

NATURAL COSINES

TABLE II

	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'	Mean				
	0°.0	0°.1	0°.2	0°.3	0°.4	0°.5	0°.6	0°.7	0°.8	0°.9	Differences				
											1'	2'	3'	4'	5'
0	1.000	1.000	1.000	1.000	1.000	1.000	.9999	9999	9999	9999	0	0	0	0	0
1	.9998	9998	9998	9997	9997	9997	9996	9996	9995	9995	0	0	0	0	0
2	.9994	9993	9993	9992	9991	9990	9990	9989	9988	9987	0	0	0	1	1
3	.9986	9985	9984	9983	9982	9981	9980	9979	9978	9977	0	0	1	1	1
4	.9976	9974	9973	9972	9971	9969	9968	9966	9965	9963	0	0	1	1	1
5	.9962	9960	9959	9957	9956	9954	9952	9951	9949	9947	0	1	1	1	2
6	.9945	9943	9942	9940	9938	9936	9934	9932	9930	9928	0	1	1	1	2
7	.9925	9923	9921	9919	9917	9914	9912	9910	9907	9905	0	1	1	2	2
8	.9903	9900	9898	9895	9893	9890	9888	9885	9882	9880	0	1	1	2	2
9	.9877	9874	9871	9869	9866	9863	9860	9857	9854	9851	0	1	1	2	2
10	.9848	9845	9842	9839	9836	9833	9829	9826	9823	9820	1	1	2	2	3
11	.9816	9813	9810	9806	9803	9799	9796	9792	9789	9785	1	1	2	2	3
12	.9781	9778	9774	9770	9767	9763	9759	9755	9751	9748	1	1	2	3	3
13	.9744	9740	9736	9732	9728	9724	9720	9715	9711	9707	1	1	2	3	3
14	.9703	9699	9694	9690	9686	9681	9677	9673	9668	9664	1	1	2	3	4
15	.9659	9655	9650	9646	9641	9636	9632	9627	9622	9617	1	2	2	3	4
16	.9613	9608	9603	9598	9593	9588	9583	9578	9573	9568	1	2	2	3	4
17	.9563	9558	9553	9548	9542	9537	9532	9527	9521	9516	1	2	3	3	4
18	.9511	9505	9500	9494	9489	9483	9478	9472	9466	9461	1	2	3	4	5
19	.9455	9449	9444	9438	9432	9426	9421	9415	9409	9403	1	2	3	4	5
20	.9397	9391	9385	9379	9373	9367	9361	9354	9348	9342	1	2	3	4	5
21	.9336	9330	9323	9317	9311	9304	9298	9291	9285	9278	1	2	3	4	5
22	.9272	9265	9259	9252	9245	9239	9232	9225	9219	9212	1	2	3	4	6
23	.9205	9198	9191	9184	9178	9171	9164	9157	9150	9143	1	2	3	5	6
24	.9135	9128	9121	9114	9107	9100	9092	9085	9078	9070	1	2	4	5	6
25	.9063	9056	9048	9041	9033	9026	9018	9011	9003	8996	1	3	4	5	6
26	.8988	8980	8973	8965	8957	8949	8942	8934	8926	8918	1	3	4	5	6
27	.8910	8902	8894	8886	8878	8870	8862	8854		8838	1	3	4	5	7
28	.8829	8821	8813	8805	8796	8788	8780	8771	8763	8755	1	3	4	6	7
29	.8746	8738	8729	8721	8712	8704	8695	8686	8678	8669	1	3	4	6	7
30	.8660	8652	8643	8634	8625	8616	8607	8599	8590	8581	1	3	4	6	7
31	.8572	8563	8554	8545	8536	8526	8517	8508	8499	8490	2	3	5	6	8
32	.8480	8471	8462	8453	8443	8434	8425	8415	8406	8396	2	3	5	6	8
33	.8387	8377	8368	8358	8348	8339	8329	8320	8310	8300	2	3	5	6	8
34	.8290	8281	8271	8261	8251	8241	8231	8221	8211	8202	2	3	5	7	8
35	.8192	8181	8171	8161	8151	8141	8131	8121	8111	8100	2	3	5	7	8
36	.8090	8080	8070	8059	8049	8039	8028	8018	8007	7997	2	3	5	7	8
37	.7986	7976	7965	7955	7944	7934	7923	7912	7902	7891	2	4	5	7	9
38	.7880	7869	7859	7848	7837	7826	7815	7804	7793	7782	2	4	5	7	9
39	.7771	7760	7749	7738	7727	7716	7705	7694	7683	7672	2	4	6	7	9
40	.7660	7649	7638	7627	7615	7604	7593	7581	7570	7559	2	4	6	8	9
41	.7547	7536	7524	7513	7501	7490	7478	7466	7455	7443	2	4	6	8	10
42	.7431	7420	7408	7396	7385	7373	7361	7349	7337	7325	2	4	6	8	10
43	.7314	7302	7290	7278	7266	7254	7242	7230	7218	7206	2	4	6	8	10
44	.7193	7181	7169	7157	7145	7133	7120	7108	7096	7083	2	4	6	8	10

NATURAL COSINES

TABLE II (Continued)

	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'	Mean				
	0°.0	0°.1	0°.2	0°.3	0°.4	0°.5	0°.6	0°.7	0°.8	0°.9	Differences				
											1'	2'	3'	4'	5'
45	.7071	7059	7046	7034	7022	7009	6997	6984	6972	6959	2	4	6	8	10
46	.6947	6934	6921	6909	6896	6884	6871	6858	6845	6833	2	4	6	8	11
47	.6820	6807	6794	6782	6769	6756	6743	6730	6717	6704	2	4	6	9	11
48	.6691	6678	6665	6652	6639	6626	6613	6600	6587	6574	2	4	7	9	11
49	.6561	6547	6534	6521	6508	6494	6481	6468	6455	6441	2	4	7	9	11
50	.6428	6414	6401	6388	6374	6361	6347	6334	6320	6307	2	4	7	9	11
51	.6293	6280	6266	6252	6239	6225	6211	6198	6184	6170	2	5	7	9	11
52	.6157	6143	6129	6115	6101	6088	6074	6060	6046	6032	2	5	7	9	11
53	.6018	6004	5990	5976	5962	5948	5934	5920	5906	5892	2	5	7	9	12
54	.5878	5864	5850	5835	5821	5807	5793	5779	5764	5750	2	5	7	9	12
55	.5736	5721	5707	5693	5678	5664	5650	5635	5621	5606	2	5	7	10	12
56	.5592	5577	5563	5548	5534	5519	5505	5490	5476	5461	2	5	7	10	12
57	.5446	5432	5417	5402	5388	5373	5358	5344	5329	5314	2	5	7	10	12
58	.5299	5284	5270	5255	5240	5225	5210	5195	5180	5165	2	5	7	10	12
59	.5150	5135	5120	5105	5090	5075	5060	5045	5030	5015	3	5	8	10	13
60	.5000	4985	4970	4955	4939	4924	4909	4894	4879	4863	3	5	8	10	13
61	.4848	4833	4818	4802	4787	4772	4756	4741	4726	4710	3	5	8	10	13
62	.4695	4679	4664	4648	4633	4617	4602	4586	4571	4555	3	5	8	10	13
63	.4540	4524	4509	4493	4478	4462	4446	4431	4415	4399	3	5	8	10	13
64	.4384	4368	4352	4337	4321	4305	4289	4274	4258	4242	3	5	8	11	13
65	.4226	4210	4195	4179	4163	4147	4131	4115	4099	4083	3	5	8	11	13
66	.4067	4051	4035	4019	4003	3987	3971	3955	3939	3923	3	5	8	11	14
67	.3907	3891	3875	3859	3843	3827	3811	3795	3778	3762	3	5	8	11	14
68	.3746	3730	3714	3697	3681	3665	3649	3633	3616	3600	3	5	8	11	14
69	.3584	3567	3551	3535	3518	3502	3486	3469	3453	3437	3	5	8	11	14
70	.3420	3404	3387	3371	3355	3338	3322	3305	3289	3272	3	5	8	11	14
71	.3256	3239	3223	3206	3190	3173	3156	3140	3123	3107	3	6	8	11	14
72	.3090	3074	3057	3040	3024	3007	2990	2974	2957	2940	3	6	8	11	14
73	.2924	2907	2890	2874	2857	2840	2823	2807	2790	2773	3	6	8	11	14
74	.2756	2740	2723	2706	2689	2672	2656	2639	2622	2605	3	6	8	11	14
75	.2588	2571	2554	2538	2521	2504	2487	2470	2453	2436	3	6	8	11	14
76	.2419	2402	2385	2368	2351	2334	2317	2300	2284	2267	3	6	8	11	14
77	.2250	2233	2215	2198	2181	2164	2147	2130	2113	2096	3	6	9	11	14
78	.2079	2062	2045	2028	2011	1994	1977	1959	1942	1925	3	6	9	11	14
79	.1908	1891	1874	1857	1840	1822	1805	1788	1771	1754	3	6	9	11	14
80	.1736	1719	1702	1685	1668	1650	1633	1616	1599	1582	3	6	9	12	14
81	.1564	1547	1530	1513	1495	1478	1461	1444	1426	1409	3	6	9	12	14
82	.1392	1374	1357	1340	1323	1305	1288	1271	1253	1236	3	6	9	12	14
83	.1219	1201	1184	1167	1149	1132	1115	1097	1080	1063	3	6	9	12	14
84	.1045	1028	1011	0993	0976	0958	0941	0924	0906	0889	3	6	9	12	14
85	.0872	0854	0837	0819	0802	0785	0767	0750	0732	0715	3	6	9	12	15
86	.0698	0680	0663	0645	0628	0610	0593	0576	0558	0541	3	6	9	12	15
87	.0523	0506	0488	0471	0454	0436	0419	0401	0384	0366	3	6	9	12	15
88	.0349	0332	0314	0297	0279	0262	0244	0227	0209	0192	3	6	9	12	15
89	.0175	0157	0140	0122	0105	0087	0070	0052	0035	0017	3	6	9	12	15
90	.0000														

NATURAL TANGENTS

TABLE III

	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'	Mean				
	0°.0	0°.1	0°.2	0°.3	0°.4	0°.5	0°.6	0°.7	0°.8	0°.9	Differences				
											1'	2'	3'	4'	5'
0	.0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	.0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	.0349	0367	0384	0402	0419	0437	0454	0472	0489	0507	3	6	9	12	15
3	.0524	0542	0559	0577	0594	0612	0629	0647	0664	0682	3	6	9	12	15
4	.0699	0717	0734	0752	0769	0787	0805	0822	0840	0857	3	6	9	12	15
5	.0875	0892	0910	0928	0945	0963	0981	0998	1016	1033	3	6	9	12	15
6	.1051	1069	1086	1104	1122	1139	1157	1175	1192	1210	3	6	9	12	15
7	.1228	1246	1263	1281	1299	1317	1334	1352	1370	1388	3	6	9	12	15
8	.1405	1423	1441	1459	1477	1495	1512	1530	1548	1566	3	6	9	12	15
9	.1584	1602	1620	1638	1655	1673	1691	1709	1727	1745	3	6	9	12	15
10	.1763	1781	1799	1817	1835	1853	1871	1890	1908	1926	3	6	9	12	15
11	.1944	1962	1980	1998	2016	2035	2053	2071	2089	2107	3	6	9	12	15
12	.2126	2144	2162	2180	2199	2217	2235	2254	2272	2290	3	6	9	12	15
13	.2309	2327	2345	2364	2382	2401	2419	2438	2456	2475	3	6	9	12	15
14	.2493	2512	2530	2549	2568	2586	2605	2623	2642	2661	3	6	9	12	16
15	.2679	2698	2717	2736	2754	2773	2792	2811	2830	2849	3	6	9	13	16
16	.2867	2886	2905	2924	2943	2962	2981	3000	3019	3038	3	6	9	13	16
17	.3057	3076	3096	3115	3134	3153	3172	3191	3211	3230	3	6	10	13	16
18	.3249	3269	3288	3307	3327	3346	3365	3385	3404	3424	3	6	10	13	16
19	.3443	3463	3482	3502	3522	3541	3561	3581	3600	3620	3	7	10	13	16
20	.3640	3659	3679	3699	3719	3739	3759	3779	3799	3819	3	7	10	13	17
21	.3839	3859	3879	3899	3919	3939	3959	3979	4000	4020	3	7	10	13	17
22	.4040	4061	4081	4101	4122	4142	4163	4183	4204	4224	3	7	10	14	17
23	.4245	4265	4286	4307	4327	4348	4369	4390	4411	4431	3	7	10	14	17
24	.4452	4473	4494	4515	4536	4557	4578	4599	4621	4642	4	7	11	14	18
25	.4663	4684	4706	4727	4748	4770	4791	4813	4834	4856	4	7	11	14	18
26	.4877	4899	4921	4942	4964	4986	5008	5029	5051	5073	4	7	11	15	18
27	.5095	5117	5139	5161	5184	5206	5228	5250	5272	5295	4	7	11	15	18
28	.5317	5340	5362	5384	5407	5430	5452	5475	5498	5520	4	8	11	15	19
29	.5543	5566	5589	5612	5635	5658	5681	5704	5727	5750	4	8	12	15	19
30	.5774	5797	5820	5844	5867	5890	5914	5938	5961	5985	4	8	12	16	20
31	.6009	6032	6056	6080	6104	6128	6152	6176	6200	6224	4	8	12	16	20
32	.6249	6273	6297	6322	6346	6371	6395	6420	6445	6469	4	8	12	16	20
33	.6494	6519	6544	6569	6594	6619	6644	6669	6694	6720	4	8	13	17	21
34	.6745	6771	6796	6822	6847	6873	6899	6924	6950	6976	4	9	13	17	21
35	.7002	7028	7054	7080	7107	7133	7159	7186	7212	7239	4	9	13	18	22
36	.7265	7292	7319	7346	7373	7400	7427	7454	7481	7508	5	9	14	18	23
37	.7536	7563	7590	7618	7646	7673	7701	7729	7757	7785	5	9	14	18	23
38	.7813	7841	7869	7898	7926	7954	7983	8012	8040	8069	5	9	14	19	24
39	.8008	8127	8156	8185	8214	8243	8273	8302	8332	8361	5	10	15	20	24
40	.8391	8421	8451	8481	8511	8541	8571	8601	8632	8662	5	10	15	20	25
41	.8693	8724	8754	8785	8816	8847	8878	8910	8941	8972	5	10	16	21	26
42	.9004	9036	9067	9099	9131	9163	9195	9228	9260	9293	5	11	16	21	27
43	.9325	9358	9391	9424	9457	9490	9523	9556	9590	9623	6	11	17	22	28
44	.9657	9691	9725	9759	9793	9827	9861	9896	9930	9965	6	11	17	23	29

NATURAL TANGENTS

TABLE III (Continued)

	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'	Mean				
	0°.0	0°.1	0°.2	0°.3	0°.4	0°.5	0°.6	0°.7	0°.8	0°.9	Differences				
											1'	2'	3'	4'	5'
45	1.0000	0035	0070	0105	0141	0176	0212	0247	0283	0319	6	12	18	24	30
46	1.0355	0392	0428	0464	0501	0538	0575	0612	0649	0686	6	12	18	25	31
47	1-0724	0761	0799	0837	0875	0913	0951	0990	1028	1067	6	13	19	25	32
48	1-1106	1145	1184	1224	1263	1303	1343	1383	1423	1463	7	13	20	27	33
49	1.1504	1544	1585	1626	1667	1708	1750	1792	1833	1875	7	14	21	28	34
50	1-1918	1960	2002	2045	2088	2131	2174	2218	2261	2305	7	14	22	29	35
51	1.2349	2393	2437	2482	2527	2572	2617	2662	2708	2753	8	15	23	30	38
52	1.2799	2846	2892	2938	2985	3032	3079	3127	3175	3222	8	16	24	31	39
53	1.3270	3319	3367	3416	3465	3514	3564	3613	3663	3713	8	16	25	33	41
54	1.3764	3814	3865	3916	3968	4019	4071	4124	4176	4229	9	17	26	34	43
55	1-4281	4335	4388	4442	4496	4550	4605	4659	4715	4770	9	18	27	36	45
56	1-4826	4882	4938	4994	5051	5108	5166	5224	5282	5340	10	19	29	38	48
57	1.5399	5458	5517	5577	5637	5697	5757	5818	5880	5941	10	20	30	40	50
58	1.6003	6066	6128	6191	6255	6319	6383	6447	6512	6577	11	21	32	43	53
59	1.6643	6709	6775	6842	6909	6977	7045	7113	7182	7251	11	23	34	45	56
60	1-7321	7391	7461	7532	7603	7.675	7747	7820	7893	7966	12	24	36	48	60
61	1.8040	8115	8190	8265	8341	8418	8495	8572	8650	8728	13	26	38	51	64
62	1.8807	8887	8967	9047	9128	9210	9292	9375	9458	9542	14	27	41	55	68
63	1.9626	9711	9797	9883	9970	2.0057	2.0145	2.0233	2.0323	2.0413	15	29	44	58	73
64	2.0503	0594	0686	0778	0872	0965	1060	1155	1251	1348	16	31	47	63	78
65	2.1445	1543	1642	1742	1842	1943	2045	2148	2251	2355	17	34	51	68	85
66	2.2460	2566	2673	2781	2889	2998	3109	3220	3332	3445	18	37	55	73	92
67	2.3559	3673	3789	3906	4023	4142	4262	4383	4504	4627	20	40	60	79	99
68	2.4751	4876	5002	5129	5257	5386	5517	5649	5782	5916	22	43	65	87	108
69	2.6051	6187	6325	6464	6605	6746	6889	7034	7179	7326	24	47	71	95	119
70	2.7475	7625	7776	7929	8083	8239	8397	8556	8716	8878	26	52	78	104	131
71	2.9042	9208	9375	9544	9714	9887	3.0061	3.0237	3.0415	3.0595	29	58	87	116	145
72	3.0777	0961	1146	1334	1524	1716	1910	2106	2305	2500	32	64	96	129	161
73	3.2709	2914	3122	3332	3544	3759	3977	4197	4420	4646	36	72	108	144	180
74	3.4874	5105	5339	5576	5816	6059	6305	6554	6806	7062	41	81	122	163	204
75	3.7321	7583	7848	8118	8391	8667	8947	9232	9520	9812	46	93	139	186	232
76	4.0108	0408	0713	1022	1335	1653	1976	2303	2635	2972	53	107	160	213	267
77	4.3315	3662	4015	4374	4737	5107	5483	5864	6252	6646					
78	4.7046	7453	7867	8288	8716	9152	9594	5.0045	5.0504	5.0970	Mean differences cease				
79	5.1446	1929	2422	2924	3435	3955	4486	5026	5578	6140	to be sufficiently accurate.				
80	5.6713	7297	7894	8502	9124	9758	6.0405	6.1066	6.1742	6.2432					
81	6.3138	3859	4596	5350	6122	6912	7720	8548	9395	7.0264					
82	7.1154	2066	3002	3%2	4947	5958	6996	8062	9158	8.0285					
83	8.1443	2636	3863	5126	6427	7769	9152	9.0579	9.2052	9.3572					
84	9.5144	9.677	9.845	10.02	10.20	10.39	10.58	10.78	10.99	11-20					
85	1143	11.66	11.91	12.16	12.43	12.71	13.00	13.30	13.62	13.95					
86	14.30	14.67	15.06	15.46	15.89	16.35	16.83	17.34	17.89	18.46					
87	19.08	19.74	20.45	21.20	22.02	22.90	23.86	24.90	26.03	27.27					
88	28.64	30.14	31.82	33.69	35.80	38.19	40.92	44.07	47.74	52.08					
89	57.29	63.66	71.62	81.85	95.49	114.6	143.2	191.0	286.5	573.0					
90	not defined														

Notes

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